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Time Series Econometrics

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Chapter 13: Vector Error Corrections

Topics:
Unit Root VARs
Triangular representation
Vector Error Correction
Testing hypotheses in the cointegration vector
Estimation of the cointegration rank

Unit Root VAR.

Consider VAR

$$Y_t = \Phi_1 Y_t + ... + \Phi_p Y_{t-p} + \alpha + \epsilon_t$$

we can rewrite this as

$$(I - \rho L)Y_t - (\zeta_1 L + ... + \zeta_{p-1} L^{p-1})\Delta Y_t = \alpha + \epsilon_t$$

where

$$\rho = \Phi_1 + ... + \Phi_p,\ \zeta_s = -(\Phi_{s+1} + ... + \Phi_p)$$

If $\rho = I$ then the vector has a unit root.
★ If $\rho = I$ and we estimate the VAR in the form

$$Y_t = \Phi_1 Y_t + ... + \Phi_p Y_{t-p} + \alpha + \epsilon_t$$

estimates $\widehat{\Phi}_k$, $\widehat{\Psi}_j$ are consistent but not asymptotically normal, so we cannot rely on standard errors.
★ If $\rho = I$ and we estimate the VAR in the form

$$\Delta Y_t = \zeta_1 \Delta Y_{t-1} + ... + \zeta_{p-1} \Delta Y_{t-p} + \alpha + \epsilon_t$$

estimates $\widehat{\Phi}_k$ are consistent and asymptotically normal, so we can rely on standard errors.

Cointegration.

Recall simple bivariate case

$$\begin{aligned} Y_{1,t} &= \beta Y_{2,t} + \epsilon_{1,t} \\ \Delta Y_{2,t} &= \epsilon_{2,t} \end{aligned}$$

We had two $I(1)$ variables with $I(0)$ linear combination $Y_{1,t} - \beta Y_{2,t}$.

If we have n variables, we may have h ($h < n$) $I(0)$ linear combinations $Y_{1,t} - \Gamma' Y_{2,t}$, where $Y_{1,t}$ is a $h \times 1$ vector, $Y_{2,t}$ is a $g \times 1$ vector, and Γ' is a $h \times g$ matrix.

Generalize the previous representation as

$$\begin{aligned} \underset{(h\times 1)}{Y_{1,t}} &= \underset{(h\times g)}{\Gamma'} \underset{(g\times 1)}{Y_{2,t}} + \underset{(h\times 1)}{\mu_1} + \underset{(h\times 1)}{z_t} \\ \underset{(g\times 1)}{\Delta Y_{2,t}} &= \underset{(g\times 1)}{\delta_2} + \underset{(g\times 1)}{u_t} \end{aligned}$$

This is known as **triangular representation**.

The number of $I(0)$ linear combinations is known as **cointegration rank**. Special cases include

$h = 0$, in which case there is no cointegration and the process should be modelled in differences;

$h = n$, in which case there any relation is $I(0)$ meaning that all the elements in Y_t are $I(0)$;

If we have a VAR, we can give a Vector ECM (VECM) representation.
Recall ECM for the simple bivariate case

$$\begin{aligned}\Delta Y_{1,t} &= -(Y_{1,t-1} - \beta Y_{2,t-2}) + \beta \Delta Y_{2,t} + \epsilon_{1,t} \\ \Delta Y_{2,t} &= \epsilon_{2,t}\end{aligned}$$

In the generic case, the VECM

$$\Delta Y_t = \zeta_1 \Delta Y_{t-1} + \ldots + \zeta_{p-1} \Delta Y_{t-p+1} + \alpha - BA' Y_{t-1} + \epsilon_t$$

where

$$A' = [I, -\Gamma']$$

Notice that BA' has rank $h < n$, so it has reduced rank.

Inference on cointegrating parameters

If we knew the cointegration rank h and if we were able to choose which variables to list under $Y_{1,t}$ and which ones in $Y_{2,t}$ in the triangular representation, we could estimate the VECM using LS (ML): estimates would be (conditionally) gaussian and standard inference would apply (notice that this is ML in the VECM, not just in the cointegrating regression of $Y_{1,t}$ on $Y_{2,t}$.

Estimation of the cointegration rank

If we do not know the cointegration rank h, we must find it out.

$n = 2$ We saw in the bivariate case that we could run the regression of $Y_{1,t}$ on $Y_{2,t}$ and test the residuals with a unit root test.

$n > 2$ But this is not practical when $n > 2$. However, we can use a likelihood ratio type test to estimate the cointegration rank for the VECM. The reduced rank matrix BA' is estimated via a technique called Reduced Rank regression.

The distribution of the likelihood ratio type statistic is very complicated, and it depends on the presence and types of deterministic terms, much like the distribution of the t statistic in the ADF unit root test. Indeed, it can be understood as a generalization of that distribution.

Robust Estimation

As the VECM is a type of VAR, we could ignore the stage of estimating the cointegration rank and estimate the VAR

$$Y_t = \Phi_1 Y_t + ... + \Phi_p Y_{t-p} + \alpha + \epsilon_t$$

instead. Estimates would be consistent, but not asymptotically normal.

On the other hand, estimating the model

$$\Delta Y_t = \zeta_1 \Delta Y_{t-1} + ... + \zeta_{p-1} \Delta Y_{t-p} + \alpha + \epsilon_t$$

then estimates would in general be inconsistent.