Lezione 5 - 07/10/2019

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ARMA(p,q)

Let ε_t w.n.$(0,\sigma^2)$, then

$$Y_t = c + \phi_1 Y_{t-1} + \ldots + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

is ARMA(p,q).

Stationarity of the whole ARMA(p,q) depends on the autoregressive part only: we have to check if the roots of

$$1 - \phi_1 z - \ldots - \phi_p z^p = 0$$

are all outside the unit circle.

For invertibility, we require that the roots of

$$1 + \theta_1 z + \ldots + \theta_q z^q = 0$$

are outside the unit circle.

ARMA(p, q)

Last week we studied MA model and AR. Excel file in Ariel and encourage us to practise.

For AR if I put -0.75 the today is the opposite of tomorrow.

AR(p)

Yt = c + Phi1 Yt-1 + … phip Yt -p + Ut

MA(q)

Xt = u + eps t + teta1 esp-1 + … + teta

How about calling Xt → Ut and put the model on AR(p) and combine the two models.

This is a marriage of the two models and these will share characteristics of both model. We learnt that MA(q) is always stationary, but AR required more than stationarity but condition on p. It's required that are all outside the unit circle.

Phi(L) Yt = c + +1 (L) epst

To have stationarity I need Phi^-1 (L) exists, so that I can write:

Yt = PHI(i)^-1 c + PHI (L) ^-1

Start from the first model (→ rossa) I rewrite using the Lag operator and line3 I collapse the polynomial with notation teta(L)

I have this two polynomial and then I say " to help stationarity I need to be able to invert polynomial of PHI".

To have stationarity we look at the moment, but there's another way look the formula on the red square. How to get to this formula? This is .. so I need that PHI(L) ^-1 exists. This mean that the fork (L) inverting the PHI(L). To have stationary I need to invert PHI(L).

LAG value of a constant is it's self, so we get PHI(1)^-1.

We so last week that we can invert the polynomial if I can break the polynomial of order p and I can invert each one of them. So the way I will be able to invert the polynomial is the same trick we were dealing the last week. To check stationarity of ARMA is the same of AR.

I manage that the stationary depdend on autoregess part only and checking the root of ths guy 1 - … are all outside the unit circle. We now know how to check for stationarity. Another was INVERTIBILITY.

How do I check for invertibility??

I do the three red line. And to have invertibility I want instead of Yt as a function of innovation, I want Yt in the function of the past.

Yt = sum Pij Yt-j + Eps t.

I want PI(L) Yt = Eps t.

Put C=0 so we don't worry about it. How do I move these guys to obtain ESPt? I need that teta(L)^-1 exist.

On past observations only. I cant take away teta(L) and past epsilon. To do it I need to invert teta(L) so that PI(L) will be PHI(L) * teta(L) ^ -1

How do I know if Teta(L) ^-1 exist? Will be the same as we did with the MA(q).

Can I get the moment? Yes, it will be painful. At least we can get the mean to know if the process is stationary. And it's the same formula of mean of AR model.

For Autocovariances: we have to establish a couple of preliminary result. 1° preliminary result we know that eps is correlated with this time and not the past time.

2° preliminary I get the expansion in the formula of ARMA(1,1) and then I look at epst-1 and the covariance. The process is stationary so if we have epst or epst-1 it's the same.

So I can now get the gamma values ➔ that are variance

For gamma2 will be very easy then

It's just phi Gamma1.

I saw the same thing on the autocovariances of AR(1). This ARMA(1,1) will look like an AR(1) for all the step after the first one. At the first steps only will look differently.

This show you this is actually correct. So dependence structure will combine both the effects.

If we go down getting autocorrelation we simply get it. If we are curious just stick some number on the excel file.

If I put -0.5 and -0.7 it will amplify the effects. So look like AR but MA will amplify the effects of AR.

In general if I have an ARMA(p, q) the autocorrelation and covariance will look like a car crash of AR and MA. But after 2 will look like and AR. It's like AR with a bit of more story.

Will be a question of this ☹ ☹ ☹

How I derive the IRF: we did it the same for AR(1).

La pagina con tante somme si può saltare. E' uguale a quella di prima ma in modo differente.

COMMON FACTORS

Excel

Put 0, 0. This is a white noise process. So let's focus on this.

Yt = Eps t → so this is a white noise. So autocorrelation does not depend on the past.

Look at ARMA(1,1)

Yt = -0.5 Yt1 + Eps + 0.5 Epst-1

Write -0.5 and 0.5

And I get the same and it's the autocorrelation of white noise. It's easy to establish that

Yt = - 0.5 L Y+1 + Eps t + 0.5 L epst and we get (1 + 0.5L) Yt -1 = (1 + 0.5 L) eps t.

We see that these factors are the same so we may simplify them.

What we get?? We get exactly the white noises that we started.

ARMA(1,1) that will look and act as it was white noises. It's because we cancelled the factors.

As long as we have a factor that are the same in the left and right we can cancel them.

So we can get the smaller model because the bigger Is called **over parametrized**. Would I want to cancel them? YES if I want to obtain white noise.

If I get ARMA(2 ,1) I can simplify in AR(1). It's simpler to manage AR(1) instead of ARMA(2,1) model.

If you do not know the parameter of the model we will no be able to estimate these guys.

These are equally valid representation of the same objects. All model are good but depends on what we want to do. MA if past innovation, representation that use the smallest number of parameter I use ARMA. But all are all the same.

Stationarity and Ergodic ARMA.

They are all stationarity and ergodic so are identically distributed white noise.

Why I like ARMA models??

- First one is sum of ARMA model we still get ARMA model. What goes for the individual goes for the community
- Second one are forecasting! The problem is that we needed to know all autocovariances, and if I had 100 observation I don't want to estimated 100 covariances. I need Teta to get all these autocovariances.

BUT I GETS BETTER

Imagine that I have AR(1).

Yt = …

What AR(1) tells you Is that Yt depend on the past (Yt-1) and Eps t is the new information that came right now. Let's think of Yt +1.

Yt+1 =

Depends on today and new information that is coming tomorrow(esp t+1). Actually, I can observer Yt, but I will never observer Eps t +1. The best option is to forecast Tt+1 | t and the value of today.

So the big formula over there will simplify in this easy formula:

Y^cappuccino t+1| t = Phi Yt

AR is good when we want to forecast because a complex formula to a very simple.

How about MA(1)? It's different.

In the MA(1)

Yt = eps + teta esp t

Tt +1 = eps t+1 + teta espt.

I got the espt but not the esp t+1. I don't observe epsilon, I only observer Y. If I had epsilon I just forecasting it right way. The problem is that I don't have it, because I have the observation (Y1, Y2…) but not eps. But If the process is invertible I can derive espsilon from the past. This suggest that I could get away with this, but I have to go back infinite to past. But what is the weight that we give to infinite past? A power of j. 0.5 ^100 is nearly 0, so instead of going all the way to infinity, after big observation we truncated it.

We assume that Eps0 = 0 so we assume that nothing happened before our first observation.

Setting – teta ^ t+1 to zero will be a very small mistakes.

Example:

AR(2) with 3 values and we see how forecast will look like.

For MA(1) let's pretend that I observe 10 observation for Yt. This will be not the true value of Eps but it's faster. It's gives me the same forecast with a error of 0.0002. The first usually is better because the second it's huge. When I have ARMA model I can put the in matrix because it's heavy. We should use the formula to get the approximate forecast knowing that estimation is pretty good. ARMA model are cool because forecast is much much easy than we saw at the very begging. Answer the two question an give me a flexible react to difficult example.