True/False Questions

1. The image of a 3x4 matrix is a subspace of R^4?

False. It is a subspace of R^3.

2. The span of vectors V_1, V_2, …,V_n consists of all linear combinations of vectors V_1, V_2, …, V_n.

True. That is the definition of the span.

3. If $V_1, V_2, \ldots, V_n$ are linearly independent vectors in R^n, then they must form a basis of R^n.

True: n linearly independent vectors in a space of dimension n form a basis.

4. There is a 5x4 matrix whose image consists of all of R^5.

False. It takes at least 5 vectors to span all of R^5.

5. The kernel of any invertible matrix consists of the zero vector only.

True. AX = 0 implies X = 0 when A is invertible.

Kernel of A = { **x** such that A**x**=**0** }

6. If 2 U + 3 V + 4 W = 5U + 6 V + 7 W, then vectors U, V, W must be linearly dependent.

True. In fact 3U+3V+3W = 0.

7. The column vectors of a 5x4 matrix must be linearly dependent.

False.

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

is an example where they are linearly independent.

8. If V_1, V_2, …, V_n and W_1, W_2, …, W_m are any two bases of a subspace ***V*** of R^{10}, then n must equal m.

True. Any two bases of the same vector space have the same number of vectors.

9. If the kernel of a matrix A consists of the zero vector only, then the column vectors of A must be linearly independent.

True. Since the kernel is zero, the columns of A must be linearly independent.

10. If the image of an nxn matrix A is all of R^n, then A must be invertible.

True. Since the columns span R^n , the matrix must have a right inverse. Since it is square, it must be invertible.

11. If vectors V_1, V_2, …, V_n span R^4 then n must be equal to 4.

False. It could be 4 or larger than 4.

12. If vectors U, V, and W are in a subspace ***V*** of R^n, then 2 U – 3 V + 4 W must be in ***V*** as well.

True. A subspace is closed under addition and scalar multiplication.

13. If a subspace ***V*** of R^n contains none of the standard vectors $E_1, E_2, \ldots, E_n$, then V consists of the zero vector only.

False. The space $\begin{bmatrix} c \\ c \\ c \end{bmatrix}$ of R^3 is a counter example.

14. If vectors V_1, V_2, V_3, V_4 are linearly independent, then vectors V_1, V_2, V_3 must be linearly independent as well.

True. Any dependence relation among V_1, V_2, V_3 can be made into a dependence relation for V_1, V_2, V_3, V_4 by adding a zero coefficient to V_4.

15. The vectors of the form $\begin{bmatrix} a \\ b \\ 0 \\ a \end{bmatrix}$

(where a and b are arbitrary real numbers) form a subspace of R^4.

True. This is closed under addition and scalar multiplication.

16. Vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ form a basis of R^3.

True. $$A\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a+2b+3c \\ b+2c \\ c \end{bmatrix}$$

For the dependence relation to equal zero, we must have c = 0, then b=0, then a=0. Thus the three vectors are linearly independent and must be a basis of R^3.

17. If two nonzero vectors are linearly dependent, then each of them is a scalar multiple of the other.

True. The dependence relation $aV+bW = 0$

has to have both a and b nonzero. Then

$V = -b/a\ W$ and $W = -a/b\ V$.

18. If V_1, V_2, V_3 are any three vectors in R^3, then there must be a linear transformation T from R^3 to R^3 such that $T(V_1) = E_1$, $T(V_2) = E_2$, and $T(V_3) = E_3$.

False. You can do this when they are independent. You cannot do it when they are dependent.

19. If A is an invertible nxn matrix, then the kernels of A and A^{-1} must be equal.

True. In fact the kernels of A and A^{-1} are both just 0.

20. For every subspace ***V*** of R^3 there is a 3x3 matrix A such that ***V*** = im(A).

True. Just pick 3 vectors which span ***V***. Use these as the columns of the matrix.