Lezione 3 - 10/10/2019

Decision problem

(Deciding if the answer for a problem is true or false)
(X , Omega, F f, D, PI)
X —> Feasible region, set of all the possible alternatives. It’s a description of all the controllable aspect (who makes the decision).

$X \subseteq \mathbb{R}^n$

Each alternative X is vector of n real number.

$X = x_i \in \mathbb{R} \ \forall i = 1, \dots, n$

X = Xi appartiene a |R per ogni i = 1.. n.
If we want to control the temperature of this room we can check the thermostat. Is this a way to describe anything? What about the possible solution of the tram way in Como. 3 alternatives considered 3 decision variable (train is 1, tram train is 2, ecc). We can describe path, vehicle and interactions as a number.
You can have a problem where X is finite or in which X is infinite. If infinite it can be discrete (enumerable) and continuous. If finite -> combinatorial (solutions are too many to be consider one by one) or strictly finite.

Omega <= R^n => sample space means the set of all possible outcome. All the uncontrollable events can be described by real numbers.

F <= R^p => indicator space and it is the set of all impacts. Means that if we consider an impact F that is described by a vector f1 to fi.
Indicator can be called objective function (we want to maximize or minimize) but for indicator is not the case.

f : X * Omega => F
This impact f that we obtain is function f evaluated by alternative x and scenario w that occurs.
f = f(x,w)

D => set of decision maker
relation between alternative and decision maker? Decision maker set alternative but how it’s work?

1) In some problem each decision maker choses a subset X(d) of the vector X.
Ex. X include three variable x1,x2,x3 and x1,x2 will be the first the of variable and then X3 is the second set of variable.
2) All agree before choosing X.
Our system can have on X preference F (simple, complex) and Omega (Certain and uncertain) and for z Decision maker (several, one)

PI : D -> 2 ^ FxF
Function where we preference function where each decision maker give me something. How can I describe the preference of single (carry before).

F and f’. What do I preferer?
<f, f’> this decision maker preferer f to f’. This decision maker also preferer f to f” and f’ to f”.

{ <f, f’> <f, f”> <f’, f”> <f,f> <f’,f’> <f”,f”>}
This is a preference relation, said with any pair with he proposes, and he postpone. It is a set of pairs. FXF are all preference and I consider a subset that is PI. Outside we consider preference not active for out decision maker. How do I write something that is a subset of something else? I used this notation <, >

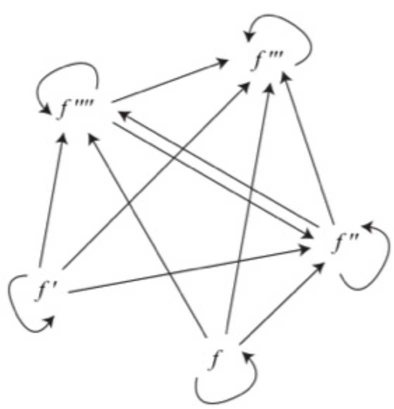
2^3^2 -> 2^9 subset pair
The idea is that the preference relation provide for each decision maker and then we find out what are the preferences. It’s a nice description 😊

(f, f’) appartiene PId   => decision maker DM prefers f to f’
When I write that f, f’ belongs to the preference of decision maker d. I mean that the decision maker D prefers impact f to impact f’. It’s a weak preference, so if I’m undecided I will prefer the same object.
f <= f’ so I accepted both <f , f’> and <f’, f>

Matrice di incidenza

	f	f'	f''	f'''	f''''
f	1	0	1	1	1
f'	1	1	1	1	1
f''	0	0	1	1	1
f'''	0	0	0	1	0
f''''	0	0	1	0	1

Graph



From a preference relation we can derivate different relations.
Indifference relation
 INDpi = { (g, g’) appartententi a FxF g is preferred by D and g’ is preferred } = { (f,f) (f’,f’) (f”, f”)}
If in the diagonal there’s 1 they are indifferent (or self loop in a graph)
(f, f’) appartenenti a INDpi -> f ~ f’

Strict preference = { (g, g’) appartenti FxF such that (:) g is preferable to g’ and g’ is not preferable to g} = { (f, f’) (f,f”) (f’, f”) }

Incompatible preference = { (g, g') appartiene FxF : g not preferred to g' and g' is not preferred to g } = 0
(No archi tra due nodi)
Clepsydra (JOIN SYMBOL) is the symbol of incompatibility

Proprieties of preference relations

Reflexibility => if you reflect the same object you get exactly the same.

$f \precsim f \quad \forall f \in F$ => tilde al posto del <=!!

Anti- symmetry

$f \precsim f' \wedge f' \precsim f \Rightarrow f' = f \quad \forall f, f' \in F$

So they are identical. If two thing are indifferent, they are the same.

Completeness

2.1.2 Proprietà di completezza

Un decisore può sempre concludere una decisione (ipotesi molto forte che talvolta porta a risultati impossibili):

$$f \not\precsim f' \Rightarrow f' \precsim f \quad \forall f, f' \in F$$

It means that we don't have incomparability. We are always able to compare two things.

Transitivity

2.1.4 Proprietà Transitiva

Solitamente i decisori non possiedono questa proprietà, anche perché è necessario modellare lo scorrere del tempo, per cui le proprietà valgono potenzialmente solo in un determinato periodo temporale. Viene generalmente considerata verificata.

$$f \precsim f' \wedge f' \precsim f'' \Rightarrow f \precsim f'' \quad \forall f, f', f'' \in F$$

If I prefer f to f' and f to f" -> I prefer f to f".

Kinds of preference relation

1) Preorder when PI = { reflexive and transitive }
2) Partial order when PI = { reflexive, transitive and antisymmetric } and they are not the same. We have to cancel e -> b arch
3) Weak order when PI = { Reflex, trans, completeness }
4) Total Order= { riflex, trans, completeness, antisymmetric}

Tabella riassuntiva

Proprietà	Preordine	Ordine debole	Ordine parziale	Ordine totale
Riflessività	X	X	X	X
Transitività	X	X	X	X
Completezza		X		X
Antisimmetria			X	X

[FOTO SUL TELEFONO] => ma anche appunti

Money pump exist in practise. Are we going to take in account time? We should write every time instant and preferences in time => could be crazy.
Paradosso del surite

No Transitivity in a loop if three nodes have only strict relation.

EXERCISES

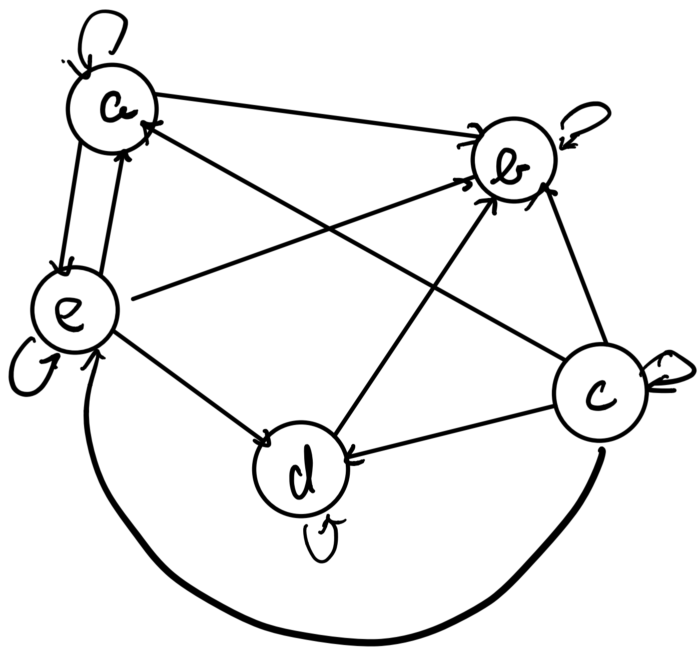
$F = \{a, b, c, d, e\}$

$R \begin{cases} (a,a)(a,b)(a,d)(a,e)(b,b)(c,a)(c,b)(c,c)(c,d)(c,e)(d,d)(d,b) \\ (e,a)(e,b)(e,d)(e,e) \end{cases}$

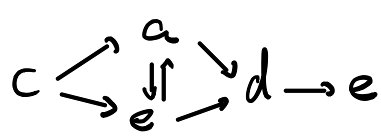
MATRIX

	a	b	c	d	e
a	1	1	0	1	1
b	0	1	0	0	0
c	1	1	1	1	1
d	0	1	0	1	0
e	0	1	0	1	1

GRAPH



- REFLEXIVE ?
  YES
- ANTI-SYMMETRIC ?
  NOPE
- COMPLETE? → ALMENO UN ARCO IN OGNI RELAZIONE TRA NODI
  YES
- TRANSITIVE ?



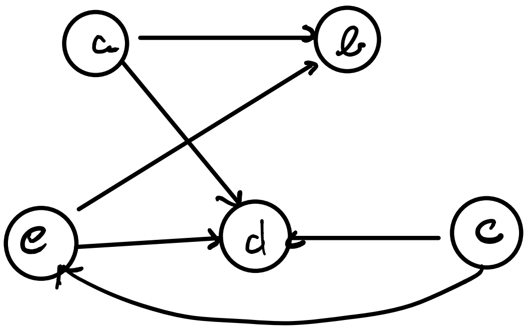
WEAK ORDER {R, C, T}

$INS = \{ (a,a)\ (b,b)\ (c,c)\ (d,d)\ (e\ e)\ (a\ e)\ (ea) \}$

WE COULD DRAW GRAPH CANCELLING ARCS WITHOUT 

$STR = \{$

ARCH WITH NO OPPOSITE



$INC = \{\ \}$

NO RELATION => NO! $\emptyset$ $\emptyset$ IN MATRIX