

Application of Vector Error Correction Model (VECM) and Impulse Response Function for Analysis Data Index of Farmers' Terms of Trade

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Abstract

Objectives: To determine the relationship among Price Index Received by Farmer (PIR), Price Index Paid by the Farmers (PIP) and the Farmers' Terms of Trade (FTT) by using the model VECM, and to attempt to know the behavior of (FTT) if there is a shock in variables PIR and PIP. **Methods/Statistical Analysis:** Vector Error Correction Model (VECM) is a model Vector Autoregressive (VAR) which can be used for data series which are non stationary and have cointegration relationship (long term relationship). The model VECM can also be used to see the movement in one variable to give a response regarding the shock produce by another variable through the graph of Impulse Response Function (IRF). **Findings:** Based on the data of Farmers' Terms of Trade in Indonesia over the periods from January 2008 to November 2013, we have determined that the best model VECM is VECM order 2 (VECM (2)). **Applications:** Based on the graph of the Impulse Response Function (IRF) we have established that the response of FTT toward the shock of a price both received and paid by the farmers is fluctuative and temporary over time.

Keywords: Farmers' Terms of Trade (FTT), Impulse Response Function, Price Index Received by Farmer (PIR), Price Index Paid by the Farmers (PIP), VAR, VECM

1. Introduction

This study involved three variables and the model VAR (Vector Autoregressive) has been used¹⁻⁶. Before the model can be chosen, the stationary data must be check. In testing the stationary data, a combination of time series plot, correlogram of ACF and unit root test can be used. The next step is to test the cointegration to analyze the long term relationship among the variables used in this study. When the data are stationary as at ordered and there is a cointegration relationship as large as r , then the model VAR which is going to be used is Vector Error Correction Model (VECM)^{2,3,5-10}. In this study the data which are going to be analyzed are Price Index Received

by Farmer (PIR), Price Index Paid by the Farmers (PIP), and Farmers' Terms of Trade (FTT). The data used in this study was obtained from BPS Statistics Indonesia (2014)¹¹ over the period from 2008 to 2013.

1.1 The Concept of Farmers' Terms of Trade (FTT)

The FTT is an indicator of the prosperity of farmers. One of the elements of the prosperity of farmers is the ability to ensure that their farm earnings can fulfil their house hold needs. The increment of prosperity can be measured from the increment ability to purchase their house hold needs. The higher the ability to purchase toward con-

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sumption needs, then the higher the Farmers' Terms of Trade (FTT). This means that the farmers will be more prosperous. Further, as an indicator of prosperity, according to Central Bureau of Statistic Indonesia (BPS), FTT can also be used in the following way: 1. To measure the change ability (terms of trade) of product sold by the farmers against the product required by the farmers in their production and household consumption needs. 2. To determine the development of the farmers' earning levels from time to time which can be used as a basis for making a policy to improve and increase the level of prosperity of farmers. 3. To demonstrate the level of competitiveness of farm products compared to other products.

1.2 The Farmers in the Concept of FTT by BPS

The farmers in the concept of FTT as defined by BPS are those farmers who work in specifically: Subsector food crops (paddy and secondary food crops; maize, soybeans, peanuts, cassava, and sweet potatoes); horticulture (vegetables, fruits plant, ornamental plants, and medicinal plants); people estate plants (coconut, coffee, clove, and tobacco); livestock (large livestock, small livestock, and the production of livestock in addition to fishery subsector (either capture fishery or aquaculture). FTT indices can be classified into two parts, indices of prices received by farmers (PIR) and indices of prices paid by farmers (PIP)¹².

1.3 Measurement of Farmers' Terms of Trade (FTT)

FTT is the comparison or ratio between the Price Index Received by Farmer (PIR) and Price Index Paid by the Farmers (PIP) which is represented in percentage. FTT is defined as follows:

$$FTT = \frac{PIR}{PIP} \times 100\%$$

Calculation of the Price Index, involved four components, namely commodities, quantity, based year, and data price.

1.4 Price Index Received by Farmer (PIR)

PIR is an index which measures the average change of price in a certain period. It is work out from a kind of package of agriculture production at the level price of producer of farmers over certain periods. PIR is calculated

by using a modified Laspeyres method¹²⁻¹⁴ and is defined as follows:

$$PIR_t = \frac{\sum_{i=1}^m \frac{Pt_{ti}}{Pt_{(t-1)i}} Pt_{(t-1)i} Q_{0i}}{\sum_{i=1}^m Pt_{0i} Q_{0i}} \times 100\%$$

where:

PIR_t = Price Index received by farmers at the t-th month.

Pt_{ti} = Price received by farmers at the t-th month for the i-th kind of goods.

$Pt_{(t-1)i}$ = Price received by farmers at the (t-1)th month for the i-th kind of goods.

$\frac{Pt_{ti}}{Pt_{(t-1)i}}$ = Relative price received by farmers at the t-th month compared with the (t-1) th month for the i-th kind of goods.

Pt_{0i} = Price received by farmers at the based year for the i-th kind of goods.

Q_{0i} = Quantity at the based year for the i-th kind of goods.

m = Number of kinds of goods that are included in the Commodities packages.

Price Index Paid by the Farmers (PIP) is an index that measures the average change of price in a certain period from a particular package commodity. It is an index which measures the average change of price during a certain period from a package commodity of goods and cost production services and increment of capital goods. In addition, it is measures household consumption expenditure in a village over particular period. PIP is defined as follows:

$$PIP_t = \frac{\sum_{i=1}^m \frac{Pb_{ti}}{Pb_{(t-1)i}} Pb_{(t-1)i} Q_{0i}}{\sum_{i=1}^m Pb_{0i} Q_{0i}} \times 100\%$$

where:

PIP_t = Price Index paid by farmers at the t-th month.

Pb_{ti} = Price paid by farmers at the t-th month for the i-th kind of goods.

$P_{b(t-1)i}$ = Price paid by farmers at the (t-1) th month for the i-th kind of goods.

$\frac{P_{b_{ti}}}{P_{b_{(t-1)i}}}$ = Relative price paid by farmers at the t-th month compared with the (t-1) th month for the i-th kind of goods.

$P_{b_{0i}}$ = Price receives by farmers during the based year for the i-th kind of goods.

Q_{0i} = Quantity at the based year for the i-th kind of goods.

m = Number of kinds of goods that included in the Commodities packages

Stationary

Many data analyses of time series are based on the assumption that the time series is stationary. The process being stationary indicate that the mean, variance and autocorrelation functions are essentially constant and do not depend on time¹⁵ that is the first two moment are time invariant⁶. If the data are nonstationary, then we need to modify the data by a certain method to make it stationary and this modification has to be done before the data are analyzed. We can modify data which are non-stationary in variance to become stationary by a particular transformation. For example, 1. If the standard deviation of a series is proportional to its level, taking the natural logarithms yields a new series with a constant variance; or 2. If the variance of the original series is proportional to its level, taking the square root induces a constant variance. Many other transformations are possible, but these two (especially the log transformation) are often useful in practice. The log transformation is both common and interpretable; the changes in a log value are relative (percent) changes in the original metric¹⁶. The logarithmic and square root transformation are a member of the family of power transformations called Box-Cox transformation^{17,18}. By this transformation we define a new series Z'_t as follows:

$$Z'_t = \frac{Z_t^\lambda - 1}{\lambda}$$

where λ is a real number. Please note that Z_t must not be negative. If some values of Z_t negative, then adding a positive constant Z_t so will ensure that all the values will

be positive¹⁶. In relation to a series which is not stationary in mean, we can usually make the data stationary by a method of differencing the data. That is, we compute the successive changes in the series for all t, as follows:

$$w_t = Z_t - Z_{t-1}$$

(If a variance stabilizing transformation has been used, we can utilize difference series Z'_t instead of Z_t .) Performing this calculation once, for all t, is known as called first differencing. If the resulting series does not yet have a constant overall mean, we then compute the first differences of the first differences for all t. That is, denote the first differences of z_t as w_t^* . Thus, the first differences of the w_t^* series are

$$w_t = w_t - w_{t-1} = (Z_t - Z_{t-1}) - (Z_{t-1} - Z_{t-2})$$

The resulting series is called the second differences of z_t . Let d denote the degree of differencing. For first differencing d= 1. For second differencing d = 2. If the original data lack a constant mean, usually setting d = 1 will create a new (differenced) series with a constant mean; setting d > 2 is almost never needed¹⁶.

1.5 Cointegration

Modeling multivariate time series data is complicated by the presence of non-stationary factors, particularly with economic data. This is due in part to the possibility of cointegration among the component series X_{it} of a non-stationary vector process X_t ^{19,20}. One possible method to deal with this problem is to difference each series until it is stationary and then fit a vector ARIMA model. However, this does not always lead to satisfactory results¹⁹. An alternative approach is to look for what is called cointegration^{3,6,7,19,21}. For example, let us suppose that X_{1t} and X_{2t} are time series data and both nonstationary; however, a particular linear combination of the two variables, say $X_{1t} - c X_{2t}$, is stationary. The two variables are then said to be co-integrated^{3,6,7,9,19}.

A more general definition of cointegration is as follows. A series X_t is said to be integrated with order d, written I(d). It needs to be differenced d times to make it stationary. If two series X_{1t} and X_{2t} are both I(d), then any linear combination of the two series will usually be I(d) as well¹⁹. However, if a linear combination exists for which the order of integration is less than d, say I(d-b), then the two series are said to be cointegrated with order (d,b) and be written as CI(d,b)^{3,6,7,19}. If this linear combination can be written in the form $\alpha^T X_t$, where $X_t^T = (X_{1t}, X_{2t})$ then

the vector α is called a cointegrating vector^{7,19}. Suppose that there are n variables series $X_{1t}, X_{2t}, \dots, X_{nt}$ as the components of vector process X_t . There is either no cointegration at all, nor is there one or two up to $n-1$ vector cointegration. If we have more than two variables, then the first step that we must carry out is to find the rank of cointegration r , namely the number of vector cointegration. To do this we can use the procedure which has been developed by Johansen²². The procedure leads to two test statistics for cointegration. The first is called the trace test, and tests the hypothesis that there are at most r cointegration vectors. The second is called the maximum Eigen value test. This procedure tests the hypothesis that there are $r+1$ cointegration vectors versus the hypothesis that there are r cointegration vectors²³.

(i) Trace test

H_0 : There exist at most r eigen values which are positive.

H_1 : There exist more than r eigen values which are positive.

$$Tr(r) = -T \sum_{i=r+1}^k \ln(1 - \hat{\lambda}_i)$$

(ii) Test λ_{\max} whether there are r or $r + 1$ vectors cointegration.

H_0 : There exist exactly r eigen values which are positive.

H_1 : There exist exactly $r + 1$ eigen values which are positive.

$$\lambda_{\max}(r, r + 1) = -T \ln(1 - \lambda_1)$$

where :

$\hat{\lambda}_i$: The estimation of Eigen values

T : Number of observations.

k : Number of endogenous variables.

This test starts from $r = 0$ and up to the first time we note that the null hypothesis is not rejected. Rank cointegration is found from the value of r . The null hypothesis is rejected for the values that are larger than the test statistics⁸.

1.6 Vector Auto Regressive (VAR)

For the analysis of data time series which involve more than one variables (multivariate time series), the Vector Autoregressive (VAR) is used^{21,24}. The structure is that

each variable comprises a linear function of past lags of itself and past lags of the other variables. For example suppose that we measure three difference time series variables, say $y_{t,1}, y_{t,2}$, and $y_{t,3}$. VAR model for order 1, VAR(1) is as follows:

$$y_{t,1} = c_1 + \phi_1 y_{t-1,1} + \phi_2 y_{t-1,2} + \phi_3 y_{t-1,3} + \varepsilon_{t,1}$$

$$y_{t,2} = c_2 + \phi_1 y_{t-1,1} + \phi_2 y_{t-1,2} + \phi_3 y_{t-1,3} + \varepsilon_{t,2}$$

$$y_{t,3} = c_3 + \phi_1 y_{t-1,1} + \phi_2 y_{t-1,2} + \phi_3 y_{t-1,3} + \varepsilon_{t,3}$$

In general, model VAR(p) for m difference time series variable can be defined as follows:

$$y_{t,i} = c_i + \sum_{j=1}^p \phi_j y_{t-j,i}$$

$$y_t = c + \sum_{i=1}^p \phi_i y_{t-1} + \varepsilon_t \quad (2.1)$$

where:

y_t : the element vector of y at time t

ϕ_i : Matrix order $n \times n$ which the elements are the coefficient of the vector y_{t-1} , for $i=1,2,\dots,p$.

p : The length of lag

c : Vector intercept

ε_t : Random vector of shock.

When the data used are stationary at the same level of differencing and there is a cointegration, then the model VAR will be combined with Error correction model to become Vector Error Correction Model (VECM)¹.

1.7 Vector Error Correction Model (VECM)

Vector Autoregressive (VAR) is one of the special forms of system simultaneous equation. Model VAR can be applied if all the variables are stationary. However, if the variables in vector Z_t are nonstationary, then the model used is Vector Error Correction Model (VECM) if there exist at least one or more cointegration relationship exists among the variables. VECM is VAR which has been designed for use with nonstationary data having cointegration relationship³.

VECM is one of the time series modelings which can directly estimate the level to which a variable can be brought back to equilibrium condition after a shock on other variables. VECM is very useful by which to estimate the short term effect for both variables and the long run effect of the time series data. The VECM(p) with the cointegration rank $r \leq k$ is as follows:

$$\Delta y_t = c + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-1} + \varepsilon_t \tag{2.2}$$

where:

Δ : Operator differencing, where $\Delta y_t = y_t - y_{t-1}$

y_{t-1} : Vector variable endogenous with the 1-st lag.

ε_t : Vector residual.

c : Vector intercept.

Π : Matrix coefficient of cointegration ($\Pi = \alpha\beta'$;

α = vector adjustment, matrix with order $(k \times r)$ and

β = vector cointegration (long-run parameter) matrix

$k \times r$)

Γ_i : Matrix with order $k \times k$ of coefficient

Endogenous of the i -th variable.

The length of the lag Optimal

To determine the length of the lag to be chosen, we can use the minimum values of the criteria. Some commonly used criteria are as follows:

(i) Final Prediction Error (FPE)

$$FPE = \frac{T+m}{T-m} \times \frac{1}{T} \sum_{t=1}^T (u_t^{(p)})^2$$

(ii) Akaike Information Criterion (AIC)

$$AIC = \ln \frac{1}{T} \sum_{t=1}^T (\hat{u}_t^{(p)})^2 + m \frac{2}{T}$$

(iii) Bayesian Criterion of Gideon Schwarz

$$SC = \ln \frac{1}{T} \sum_{t=1}^T (\hat{u}_t^{(p)})^2 + m \frac{\ln T}{T}$$

(iv) Hannan-Quinn Criterion

$$HQ = \ln \frac{1}{T} \sum_{t=1}^T (\hat{u}_t^{(p)})^2 + m \frac{2 \ln(\ln T)}{T}$$

Where $\hat{u}_t^{(p)}$ are denotes the residuals estimation from the model VAR(p), m is number of dependent variables, T is number of observations and p is the length of model VAR⁸.

1.8 Testing for Normality

It is a standard tool to conduct a diagnostic check to identify a model before it can be used for forecasting^{25, 26}. Testing for normality of residual is a test designed to determine the normality residual of data. The purpose

of this test is to ascertain whether the residuals from the data are normally distributed or not. To testing for normality, we can use the Jarque-Bera (JB) Test of Normality. This test used the measure of skewness and kurtosis. In its application to decide whether the null hypothesis is rejected or not, we compare the value of Jarque-Bera (JB) with the value of chi-square (χ^2) with 2 degrees of freedom. The calculation of JB is as follows:

$$JB = \frac{n}{6} \left(S^2 + \frac{(k-3)^2}{4} \right)$$

where :

n : Number of sample

S : Expected Skewness

$$S = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^3}{\left(\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \right)^{\frac{3}{2}}}$$

k : Expected Excess Kurtosis

$$k = \frac{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^4}{\left(\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2 \right)^2}$$

Jarque-Bera (JB) (which is used in testing for normality for residuals) determined that the calculation used is as follows:

$$JB = \frac{n-k}{6} \left(S^2 + \frac{(K-3)^2}{4} \right)$$

where :

k : Number of independent variables.

1.9 Testing for Stability

The stability system VAR can be from the inverse roots characteristics polynomial of AR. A VAR system is said to be stable (stationary) if all roots have a modulus of less than one and all are contained within the unit circle. According to Lutkepohl⁹ that the model VAR(p) given in Equation (2.1) can be written as follows:

$$y_t = c + \varnothing_1 y_{t-1} + \dots + \varnothing_p y_{t-p} + \varepsilon_t \tag{2.3}$$

If this mechanism is started at certain time, for example at $t=1$, then we have :

$$\begin{aligned}
 y_1 &= c + \phi_1 y_0 + \varepsilon_1; \\
 y_2 &= c + \phi_1 y_1 + \varepsilon_2 \\
 &= c + \phi_1 (c + \phi_1 y_0 + \varepsilon_1) + \varepsilon_2 \\
 &= (I_k + \phi_1) c + \phi_1^2 y_0 + \phi_1 \varepsilon_1 + \varepsilon_2 \\
 y &= (I_k + \phi_1 + \dots + \phi_1^{t-1}) c + \phi_1^t y_0 + \sum_{i=0}^{t-1} \phi_1^i \varepsilon_{t-i} \\
 y_t &= (I_k + \phi_1 + \dots + \phi_1^{t-1}) c + \phi_1^t y_0 + \sum_{i=0}^{t-1} \phi_1^i \varepsilon_{t-i}
 \end{aligned} \tag{2.4}$$

Therefore vector (y_1, y_2, \dots, y_t) can be determine by vector $(y_0, \varepsilon_1, \dots, \varepsilon_t)$ and The joint distribution of y is determined by joint distribution of (y_1, y_2, \dots, y_t) is determined by joint distribution of $(y_0, \varepsilon_1, \dots, \varepsilon_t)$.

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t = (I_k + \phi_1 + \dots + \phi_1^{t-1}) c + \phi_1^t y_0 + \sum_{i=0}^{t-1} \phi_1^i \varepsilon_{t-i} \tag{2.5}$$

If all the eigen values of ϕ_1 are less than 1 in absolute values, then the order of $\phi_1^i, i=0,1,2,\dots$ is summable. And the model y_t is stochastic process and defined as:

$$y_t = \mu + \sum_{i=0}^{\infty} \phi_1^i \varepsilon_{t-i}, t = \dots -1, 0, 1, \dots \tag{2.6}$$

Given the definition of characteristics polynomial of a matrix we call this polynomial the reverse characteristic polynomial of the VAR(p) process. Hence, the Process (2.3) is stable if the reverse characteristics polynomial has no roots in or on the complex unit circle. Formally Y_t can be said to stable if

$$\det(I_{kp} - \phi z) = \det(I_k - \phi_1 z - \dots - \phi_p z^p) \neq 0 \text{ for } |z| \leq 1 \tag{2.7}$$

This condition is called the stability condition².

1.10 Impulse Response Function (IRF)

In²⁷ state that the IRF is a method that can be used to determine the response of an endogenous variable toward a shock from the other variables. A Vector Autoregressive can be written as the form of Vector Moving Average (VMA). The representation of VMA is an important feature which enables us to see the various shocks on vari-

able in the VAR model. As an illustration, we used two variables in the form of matrix VAR³ as follows:

$$y_t = b_{10} - b_{12} z_t + \alpha_{11} y_{t-1} + \alpha_{12} z_{t-1} + \varepsilon_{yt} \tag{2.8}$$

$$z_t = b_{20} - b_{21} y_t + \alpha_{21} y_{t-1} + \alpha_{22} z_{t-1} + \varepsilon_{zt}$$

In matrix notation it can be written as

$$\begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix} \begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix}$$

or

$$B x_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t \tag{2.9}$$

where

$$B = \begin{pmatrix} 1 & b_{12} \\ b_{21} & 1 \end{pmatrix}, x_t = \begin{pmatrix} y_t \\ z_t \end{pmatrix}, \Gamma_0 = \begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix},$$

$$\Gamma_1 = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \text{ and } \varepsilon_t = \begin{pmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{pmatrix}$$

By using the equation model VAR given in (2.1) the general form which is assumed has a stable condition is as follows:

$$X_t = \mu + \sum_{i=0}^{\infty} A_1^i e_{t-1}$$

where:

$$X_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}, \mu = \begin{bmatrix} y \\ z \end{bmatrix} \text{ and } A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

We have:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{bmatrix} e_{1t-1} \\ e_{2t-1} \end{bmatrix} \tag{2.10}$$

Equation (2.10) states that Y_t and Z_t in terms of the order $\{e_{1t}\}$ and $\{e_{2t}\}$ which can be written as $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$. Premultiplication Equation (2.9) by B^{-1} which enables us to have the model VAR in the form

$$X_t = A_0 + A_1 X_{t-1} + e_t$$

where

$$A_0 = B^{-1} \Gamma_0, A_1 = B^{-1} \Gamma_1 \text{ and } e_t = B^{-1} \varepsilon_t.$$

It is noted that the term error (error e_t) refers to the combination of shocks (c_t). By using the Equation $e_t = B^{-1} \varepsilon_t$, then e_{1t} and e_{2t} at the Equation (2.3) can be written as:

$$e_{1t} = \frac{(\varepsilon_{yt} - b_{12}\varepsilon_{zt})}{(1 - b_{12}b_{21})} \text{ and } e_{2t} = \frac{(\varepsilon_{zt} - b_{12}\varepsilon_{yt})}{(1 - b_{12}b_{21})}$$

The equation above can be written in the matrix form as follows:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \quad (2.11)$$

The Equations (2.10) and (2.11) can be combined in the following form:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \frac{1}{1 - b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{pmatrix}$$

The notation above can be simplified by defined matrix Φ of order 2×2 . Then the representation of VMA at the Equations (2.10) and (2.11) can be written in the form $\{\varepsilon_{yt}\}$ and $\{\varepsilon_{zt}\}$:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{pmatrix} \Phi_{11}(i) & \Phi_{12}(i) \\ \Phi_{21}(i) & \Phi_{22}(i) \end{pmatrix} \begin{pmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{pmatrix} \quad (2.12)$$

with the element $\Phi_{jk}(i)$:

$$\Phi_i = \frac{1}{1 - b_{12}b_{21}} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^i \begin{pmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{pmatrix}$$

Equation (2.12) can be written in the form x_t as:

$$x_t = \mu + \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i} \quad (2.13)$$

The coefficients $\Phi_{11}(i)$, $\Phi_{12}(i)$, $\Phi_{21}(i)$ and $\Phi_{22}(i)$ are called *impulse response functions*. The plot of impulse response function (is the plot of $\Phi_{jk}(i)$ and i) is a practical way to visualize the behaviour of $\{yt\}$ and $\{zt\}$ in response toward the shocks (*shocks*)³.

2. Method

Step 1: Identification

At this step, we identify and check whether the time series data are stationary. If the plot of time series data moves around a constant and has no trend, then we can say that the data are stationary in mean. But if the data are moving fluctuatively and are not constant, then we say that the data are not stationary in variance. To make the data

stationary in variance we can use Box- Cox transformation^{6,16}. If the stationary in variance has been attained, but the data still has a trend, then we can use the *differencing* process to make the data stationary in mean^{6,16}.

Step 2: Estimation of the Model

If the data fulfil the assumption of stationary in mean and variance, then we can test the order of cointegration by using Johansen's test^{5,6,10,16}. Then we can perform calculations to determine the length of optimal lag p by using the minimum values of the information criteria given in^{2,8,16}. At this step we will find the estimator of parameters by using the maximum likelihood method.

Step 3: Testing for Residual

The model we found in Step 2 needs to be checked against the normality of the residuals and testing the stability. If the residuals are normally distributed and have high stability, then the model can be used⁹.

Step 4: IRF Analysis

Impulse Response Function (IRF) is used to depict how the rate of a shock for a variable reacts toward the response of others variables. It also attempts to determine the length of the impact of the shock from one variable to the other variables^{2,6,8}.

3. Results and Discussion

3.1 Identification

The first step of modeling time series is to check whether or not the time series data are stationary. To check the stationary of the data we can use time series plot, correlogram ACF and unit root test.

Plot time series

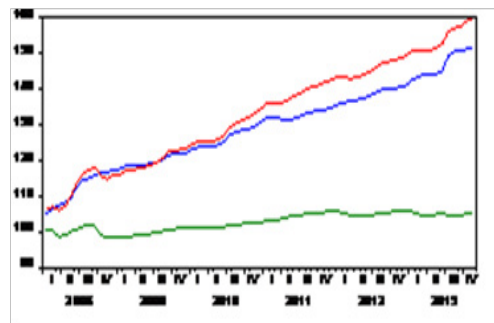


Figure 1. Plot time series data FTT.

Figure 1 shows that the three variables are stationary in variances, but not in means because the graph shows that there are trends.

Correlogram ACF

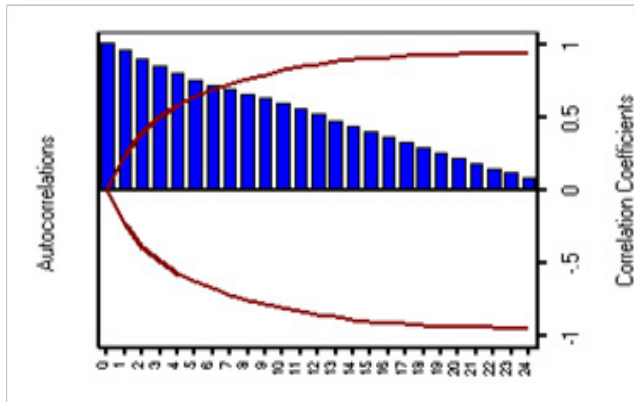


Figure 2. ACF for variable PIR.

Unit root test

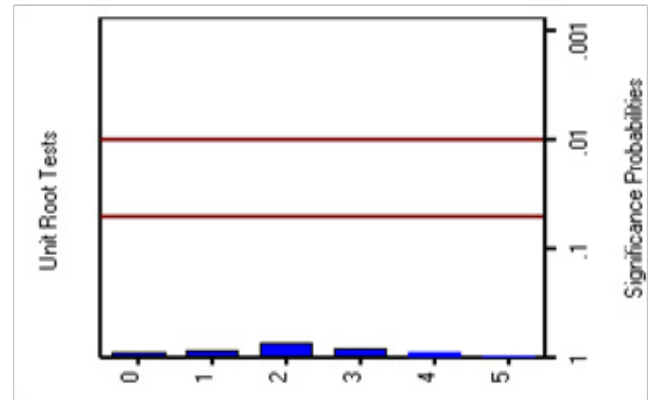


Figure 5. Unit root test variable PIR.

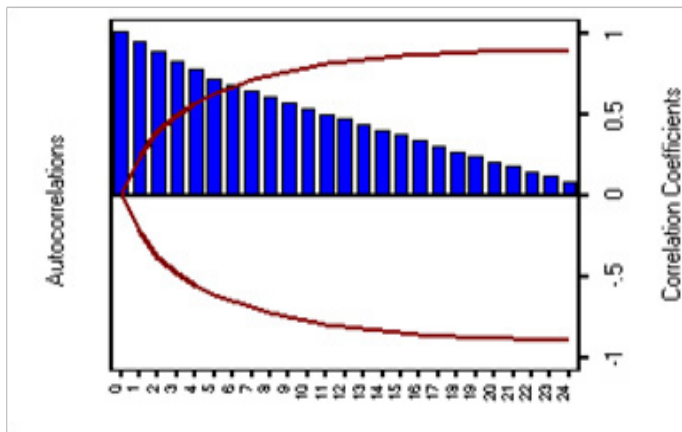


Figure 3. ACF for variable PIP.

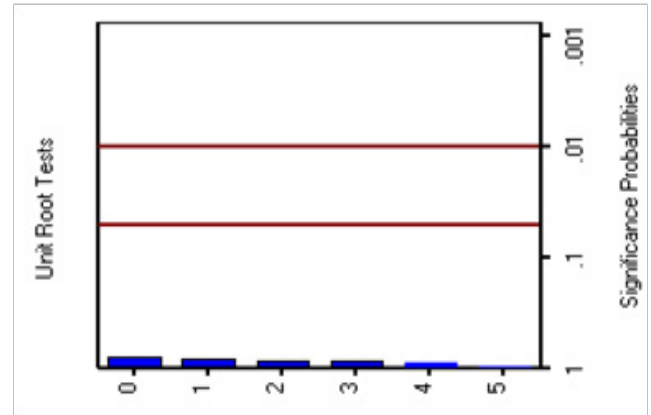


Figure 6. Unit root test variable PIP.

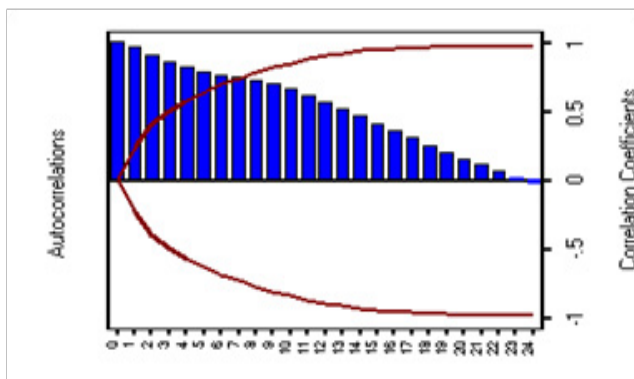


Figure 4. ACF for variable FTT.

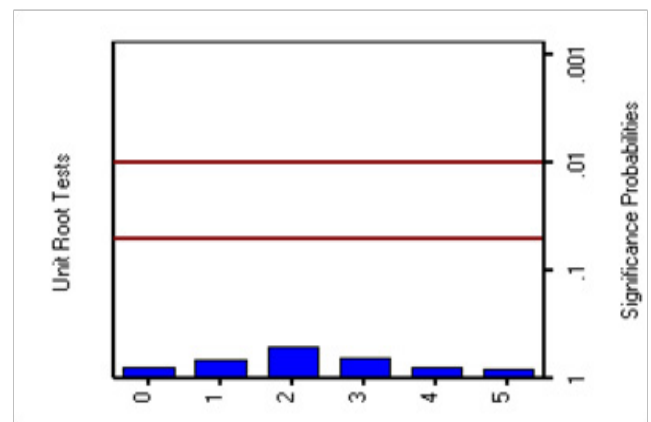


Figure 7. Unit root test variable FTT.

Figure 2, 3 and 4 shows that from lag 1 to lag 2, there is a slow decrease tending to zero. This means that the coefficient of correlation difference significantly from zero. Accordingly, Then we can conclude that, based on correlogram ACF, the three variables of data FTT are not stationary.

Hypothesis:

H_0 = Data FTT is nonstationary

H_1 = Data FTT stationary

Figure 5, 6, and 7 shows that, at any lag, the three variables do not pass through the significance $\alpha = 0.05$,

this means that the p-values of lag 0 to lag 5 are greater than 0.05. Thus, it is not sufficient evidence to reject H_0 , so we can conclude that the data FTT are nonstationary. From the plot of time series, we know that the data FTT is stationary in variance. This also can be shown through Box-Cox transformation to determine that the best value λ (lambda). By using SAS program, the best value of λ is as follows:

Table 1. The value of λ from Box-Cox transformation for variable PIR

Transformation Information for BoxCox(PIR)			
Lambda	R-Square	Log Like	Transformation
-1.000	0.99	-32.9417	Z_1
-0.500	0.99	-11.7695	Z_2
0.000	1.00	17.6246	$\ln Z_3$
0.050	1.00	21.3193	Z_4

Table 2. The value of λ from Box-Cox transformation for variable PIP

Transformation Information for BoxCox(PIP)			
Lambda	R-Square	Log Like	Transformation
-1.000	0.99	-13.6807	Z_1
-0.500	0.99	7.3444	Z_2
0.000	1.00	36.6248	$\ln Z_3$
0.050	1.00	40.3099	Z_4

Tables 1, 2 and 3 show that the values of λ which can be used are = 1, for Z_1 transformation. With the level of significance of 95% it can be concluded that the data are stationary in variances. Next, in order to make the data are stationary in means, we need to perform differencing on data which have been transformed by Box-Cox transformation with $\lambda = 1$. After the first differencing, then the stationary data can be rechecked through time series plot, correlogram ACF and unit root test. Plot time series

Table 3. The value of λ from Box-Cox transformation for variable FTT

Transformation Information for BoxCox(FTT)			
Lambda	R-Square	Log Like	Transformation
-1.000	0.99	223.6327	Z_1
-0.500	0.99	244.2874	Z_2
0.000	1.00	273.3005	$\ln Z_3$

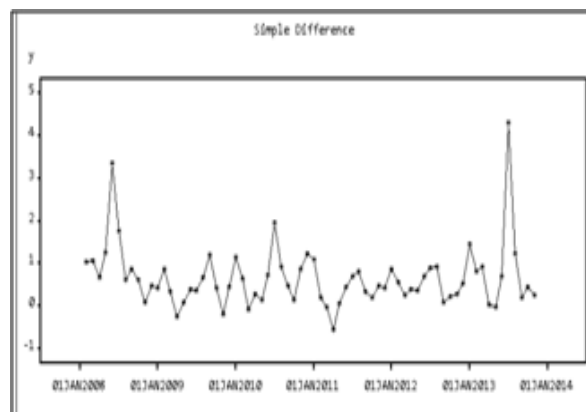


Figure 8. Plot time series PIR after the first differencing.

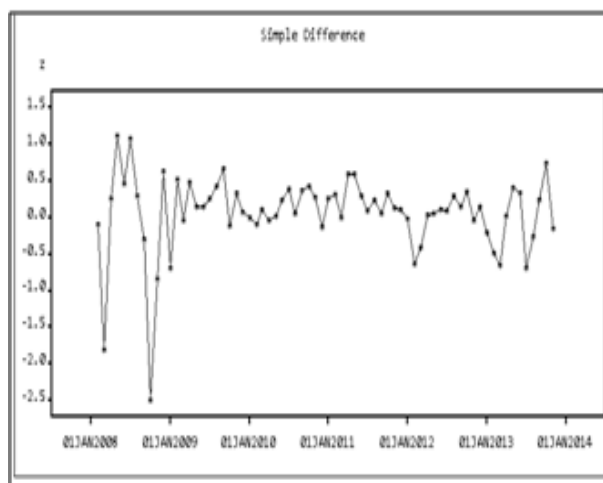


Figure 9. Plot time series PIP after the first differencing.

Figures 8, 9 and 10 are time series plot data which have been transformed by using Box-Cox transformation with $\lambda = 1$. The first differencing shows that the data are stationary.

Correlogram ACF

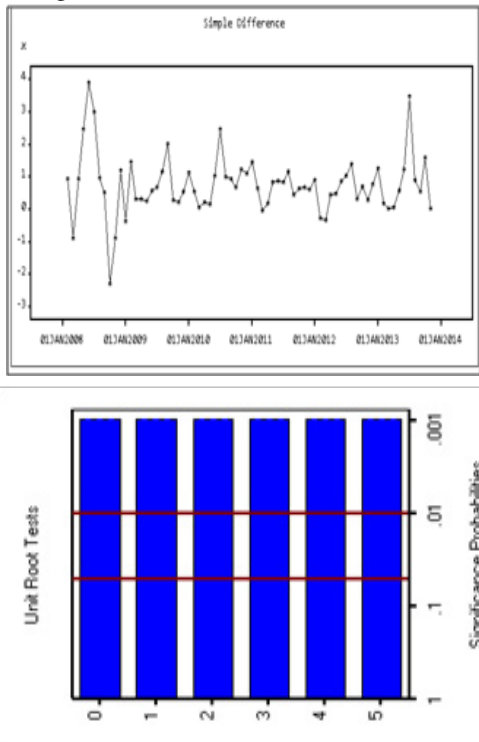


Figure 10. Plot time series FTT after the first differencing.

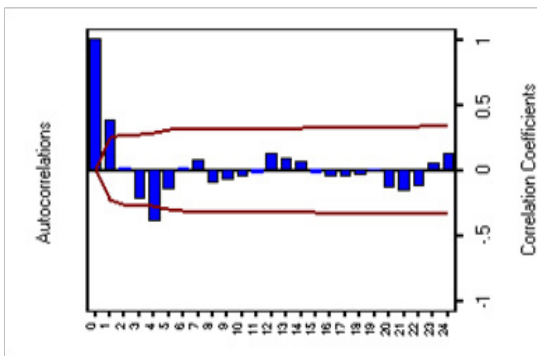


Figure 11. ACF variable PIR Box-Cox(1) after first differencing.

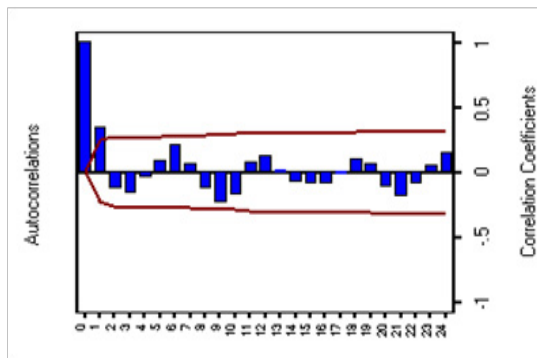


Figure 12. ACF variable PIP Box-Cox(1) after first differencing.

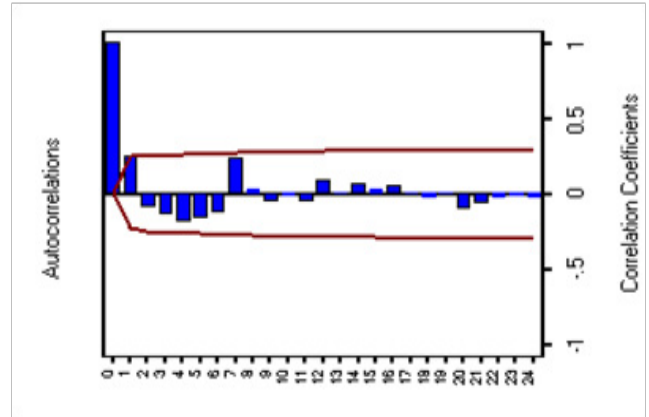


Figure 13. ACF variable FTT Box-Cox(1) after first differencing.

Figures 11, 12 and 13 shows that from lag 1 to lag 2 and up to lag 24 decreases tend to zero. Thus, we can conclude that based on correlogram ACF, the three variables data FTT after the differencing are stationary.

Unit root test

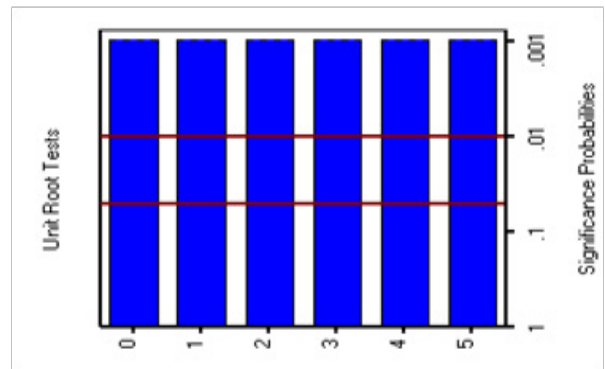


Figure 14. Unit root test variable PIR Box-Cox(1) after first differencing.

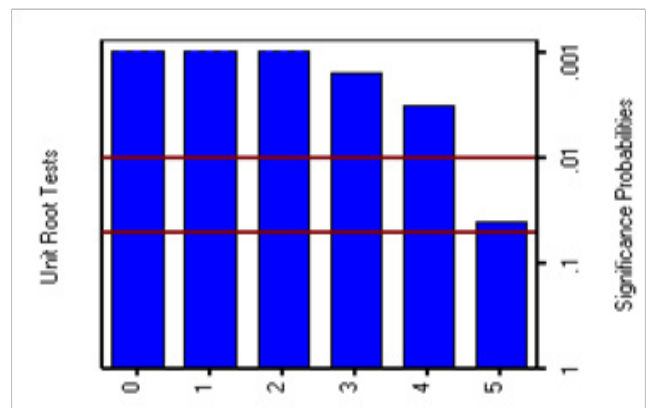


Figure 15. Unit root test variable PIP Box-Cox(1) after first differencing.

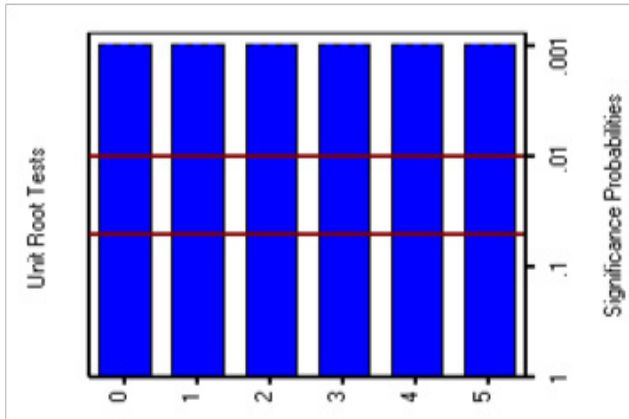


Figure 16. Unit root test variable FTT Box-Cox(1) after first differencing.

Hypothesis :

H_0 = Data FTT is not stationary after first differencing

H_1 = Data FTT is stationary after first differencing

Figures 14, 15 and 16 show that for any lag for the three variables FTT after the first differencing passes through the significant level $\alpha = 0.05$. This means that the p-value at lag 0 to lag 5 is less than 0.05. The H_0 is then rejected and we conclude that the data are stationary.

3.2 Cointegration Test

If the nonstationary data become stationary at the first differencing, then there is a high probability that they have a cointegration relationship (long term relationship) among the variables. To establish whether or not there is a cointegration relationship or not we can use Johansen's test^{28,29}.

Table 4. Results of cointegration test by using Johansen test

Cointegration Rank Test Using Trace			
H_0 : Rank=r	H_1 : Rank>r	Trace (λ)	5% Critical Value
0	0	64.7570	24.08
1	1	8.4816	12.21
2	2	3.7395	4.14

H_0 is not rejected if the value $\lambda_{trace} < \text{critical values}$. Table 4. $\lambda_{trace} < \text{Critical values}$ when $r = 1$. Thus, we can conclude that the variables at FTT have cointegration at rank = 1. VAR model is used when one or all of the variables of time series data are stationary, while VECM model is used when all the variables used are nonstationary. If the variables PIR, PIP and FTT in the data FTT are nonstationary it has also been proved that they have a cointegration relationship among the variables. In this instance the model VAR (p) which is used is VECM (p) model^{29,30}.

3.3 Model Estimation

The first step to be taken is the VECM model to determine the optimum lag by comparing every lag to the criteria used.

In Table 5 the minimum values from each of the information criteria are given with star sign (*). The table above indicate that the lag optimal is at lag 2, hence, the VECM(p) model which is used is VECM(2). The next step is to estimate the parameters in the model. The estimation parameters are given in the Table 6.

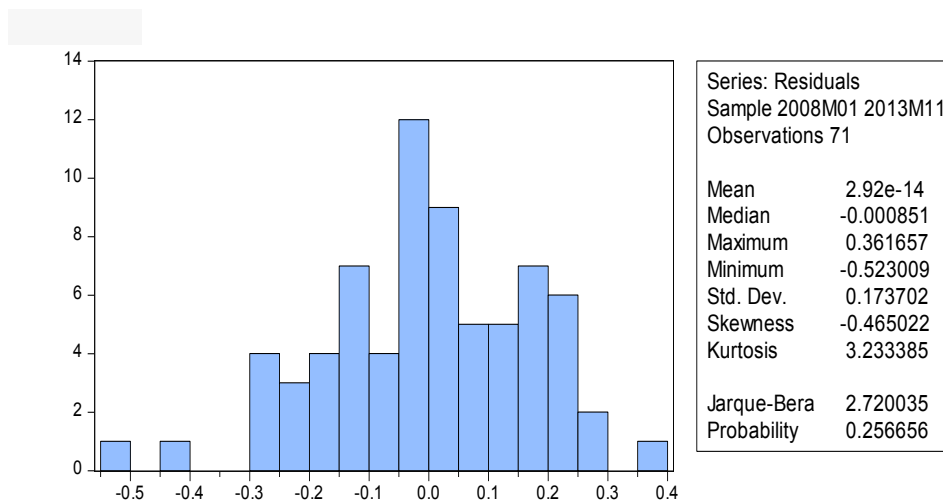


Figure 17. Histogram residual and the value of Jarque-Bera test of normality.

Table 5. The result of VAR order selection criteria

VAR Lag Order Selection Criteria					
Lag	LR	FPE	AIC	SC	HQ
0	NA	3.724	9.828	9.927	9.867
1	572.97	0.00055	1.00214	1.3970*	1.15839
2	24.80*	0.00047*	0.85736*	1.54838	1.13079*
3	8.800	0.00053	0.97163	1.95880	1.36226
4	5.114	0.00064	1.14557	2.42890	1.65339

Table 6 demonstrates that the model VECM (2) is as follows:

$$\Delta Y_t = -26.037 + \begin{pmatrix} 0.831 & -0.869 & -1.008 \\ 1.143 & -1.196 & -1.386 \\ -0.206 & 0.215 & 0.249 \end{pmatrix} Y_{t-1} + \begin{pmatrix} 0.124 & 0.135 & 0.505 \\ 0.213 & 0.119 & 0.126 \\ -0.233 & 0.183 & 0.498 \end{pmatrix} \Delta Y_{t-1} + \varepsilon_t$$

Residual Test

Normality test

Hypothesis:

H₀ = residuals are normally distributed

H₁ = residual are not normally distributed

If the statistic $JB < x^2_{(\alpha,2)}$ orp-value $> \alpha$, then Ho is not rejected which means that the assumption normality is satisfied.

Based on Figure 17, $JB = 2.720035$ and the critical value of chi square with 2 degrees of freedom and the level of significant 0.05 is $x^2_{(0.05,2)} = 5.99$. Since $JB < x^2_{(\alpha,2)}$, then we do not reject Ho. Therefore, we conclude that the residual are normally distributed with level of significance of 95%.

Stability Test Model VECM is said to have high stability when the characteristic polynomial of AR has modulus ≤ 1 .

Table 7 shows that the modulus of the characteristics roots at all lag are ≤ 1 . Thus, we can conclude that the model VECM (2) is appropriate to be used since it has high stability.

4. Impulse Response Function

One of the advantages of the application of VAR model is the ability to analyze the response of a variable toward a shock or a change in the other variable to the variable itself. To determine the behavior of a variable in response

Table 6. Estimation parameters for model VECM(2) Model Parameter Estimates

Model Parameter Estimate					
	Parameter	Estimate	Error	T-value	Variable
D_ PIR	CONST1	105.98548	67.39451	3.57	1
	AR1_1_1	0.83068	0.53081		PIR(t-1)
	AR1_1_2	-0.86951	0.55563		PIP(t-1)
	AR1_1_3	-1.00001	0.64413		FTT(t-1)
	AR2_1_1	0.12395	1.73268	0.07	D_PIR(t-1)
	AR2_1_2	0.13530	1.72121	0.08	D_PIP(t-1)
	AR2_1_3	0.50537	2.06749	0.24	D_FTT(t-1)
D_ PIP	CONST1	145.44184	50.35968	2.89	1
	AR1_2_1	1.14258	0.39664		PIR(t-1)
	AR1_2_2	-1.19599	0.41518		PIP(t-1)
	AR1_2_3	-1.38650	0.48132		FTT(t-1)
	AR2_2_1	0.21277	1.29472	0.16	D_PIR(t-1)
	AR2_2_2	0.11931	1.28615	0.09	D_PIP(t-1)
	AR2_2_3	0.12648	1.54491	0.08	D_FTT(t-1)
D_ FTT	CONST1	-26.03753	42.09219	-0.62	1
	AR1_3_1	-0.20583	0.33153		PIR(t-1)
	AR1_3_2	0.21545	0.34702		PIP(t-1)
	AR1_3_3	0.24976	0.40230		FTT(t-1)
	AR2_3_1	-0.23291	1.08217	-0.22	D_PIR(t-1)
	AR2_3_2	0.18344	1.07500	0.17	D_PIP(t-1)
	AR2_3_3	0.49797	1.29128	0.39	D_FTT(t-1)

to a shock we can use the graph of Impulse Response Function (IRF). Analysis of IRF is conducted by providing a plot from impulse response function (namely the coefficient ϕ_i) to visualize the change of response of farmers' exchange values toward the shock experienced due to the change of price received and price paid by farmers.

Table 7. Characteristics roots of AR

Roots of AR Characteristics Polynomial					
Index	Real	Imaginary	Modulus	Radian	Degree
1	1.00000	0.00000	1.0000	0.0000	0.00000
2	1.00000	0.00000	1.0000	0.0000	0.00000
3	0.80447	0.00000	0.8045	0.0000	0.00000
4	0.30539	0.00000	0.3054	0.0000	0.00000
5	0.25792	0.12092	0.2849	0.4384	25.1194
6	0.25792	-0.12092	0.2849	-0.4384	-25.1194

$$x_t = \mu + \sum_{i=0}^{40} \phi_i \varepsilon_{t-i}$$

$$FTT_t = \mu + \sum_{i=1}^{40} \varphi_i \varepsilon_{t-i}$$

The changes in prices which are received by farmers can be attributed to varying factors, namely the price of fertilizer and pesticide, the quantity of production or the season. Figure 18 shows that when a shock occurs of one standard deviation at the first month from the variable price received by farmers it has a positive impact as large as 0.306520 toward the change of farmers' exchange values. This positive impact continues up to the peak at the second month and proceeds to move up and down up to the 13-th month. Following this the movement becomes an equilibrium condition.

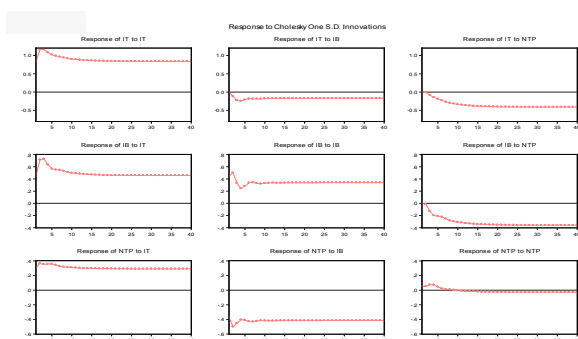


Figure 18. Graph of response of farmers' exchange values toward the change of price received by farmers (PIR).

The reasons for change in prices paid by the farmers are due to a number of factors including change in govern-

ment policy toward the production costs, political factors as well as change in currency values. Based on Figure 19, it can be seen that when there is a change in price paid by farmers, the variable farmers exchange values provide negative responses. A shock in the first month with the variable farmers' exchange values provide a response as large as -0.376014. This negative impact has peaked in the second period and slowly change to increase up to the 9-th period and move to the equilibrium condition.

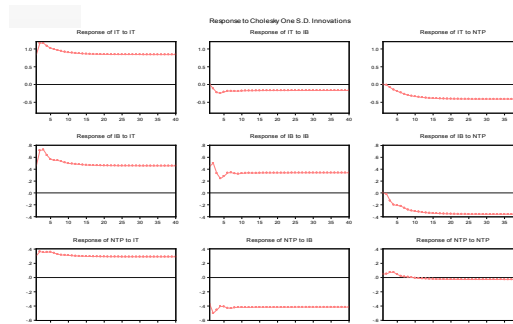


Figure 19. Graph of response farmers' exchange values toward the change of price paid by farmers (PIP).

Based on Figure 19, the response towards farmers' exchange values received and paid by farmers' shows the equilibrium movement, but tends to be not close to zero. This means that after it attains the level of equilibrium, the changes between the price received by farmers and the price paid by farmers will be responded to by permanent changes to the farmers' terms of trade values.

5. Conclusion

Based on the discussion and results detailed above, we can conclude that the data for Farmers' Terms of Trade (FTT), Price Index Received by farmers (PIR), and the Price Index Paid by farmers (PIP) can be modeled by using Vector Error Correction Model (2), VECM (2). By using this model, it was found that the Farmers' Terms of Trade (FTT), Price Index Received by farmers (PIR), and Price Index Paid by farmers (PIP) they have a cointegration relationship at rank = 1. By using Impulse Response Function (IRF) it was found that when the price paid by farmers changed, then the Farmers' Terms of Trade provide negative responses (the opposite direction). On the other hand, if the price received by farmers changed, then the farmers' terms of trade offers a positive response (the same direction). Thus, the proportion of shock for

the changed in prices paid by farmers did not have a high contribution (effect) upon the farmers' terms of trade. On the other hand, the proportion of shock towards the changed in the price received by farmers has a high contribution (effect) upon the farmers' terms of trade.

6. Acknowledgement

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