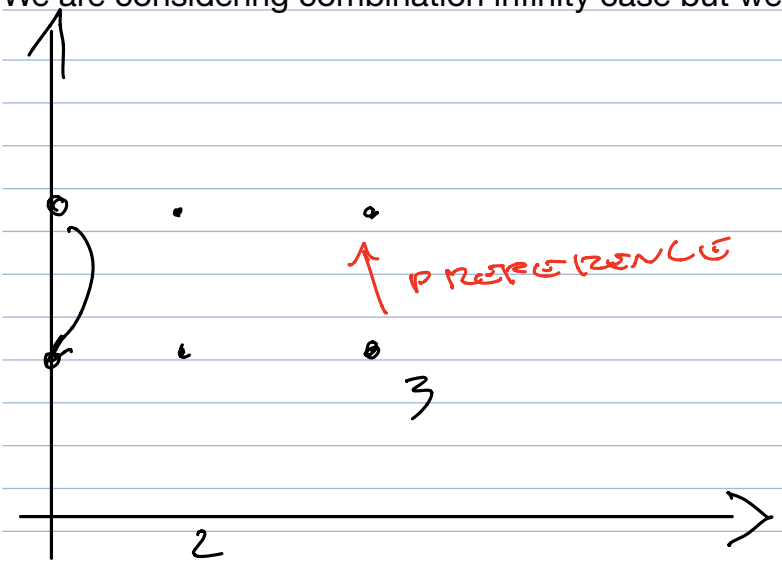
Multi attribute utility theory

$\tilde{\succ}$ Weak Order —> $u(f)$ Consistent with $\tilde{\succ}$

$$f \succeq f' \iff u(f) \geq u(f')$$

We are considering combination infinity case but we are considering continuous

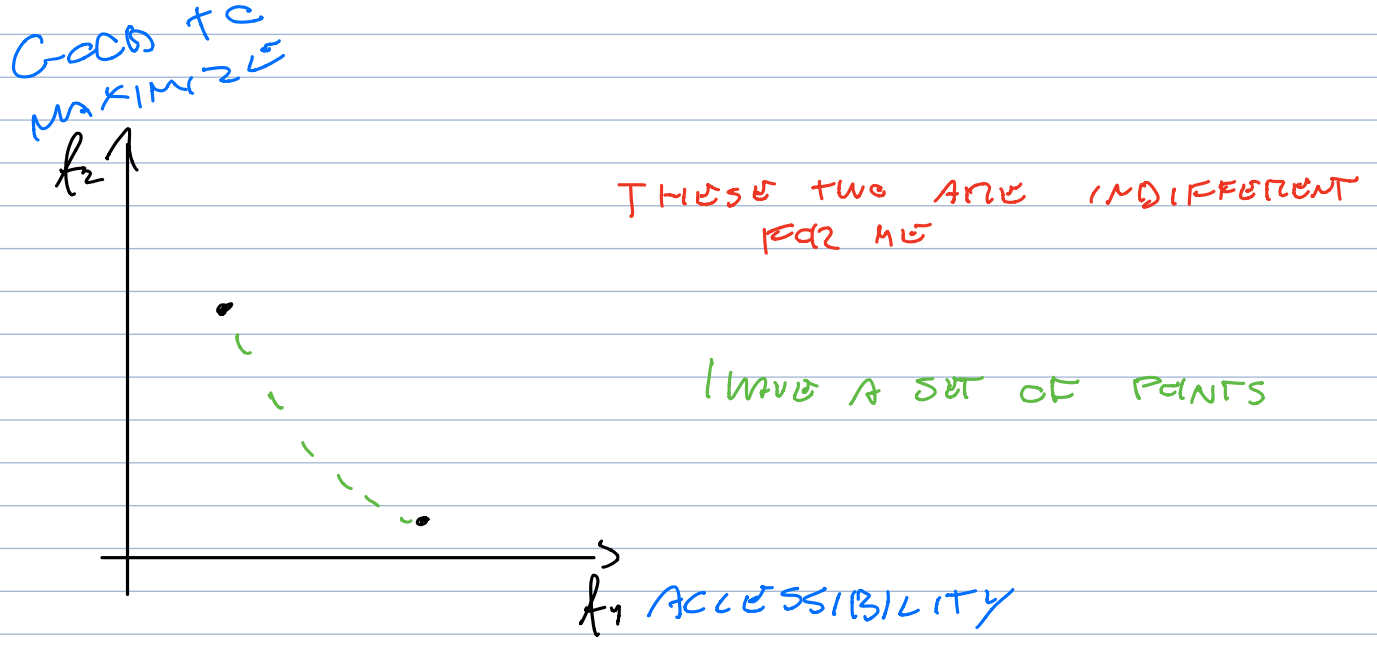


X CONTINUOS

F CONTINUOUS

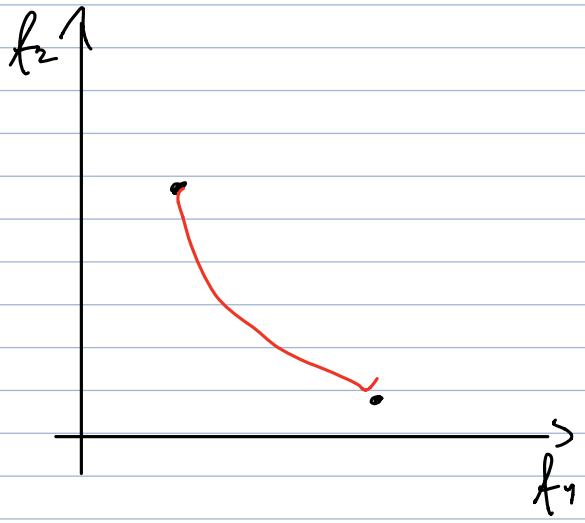
Indifference curve

Subset of reciprocally indifferent



If we have regular relation
If i change f1 not very much, indifference will not change very much

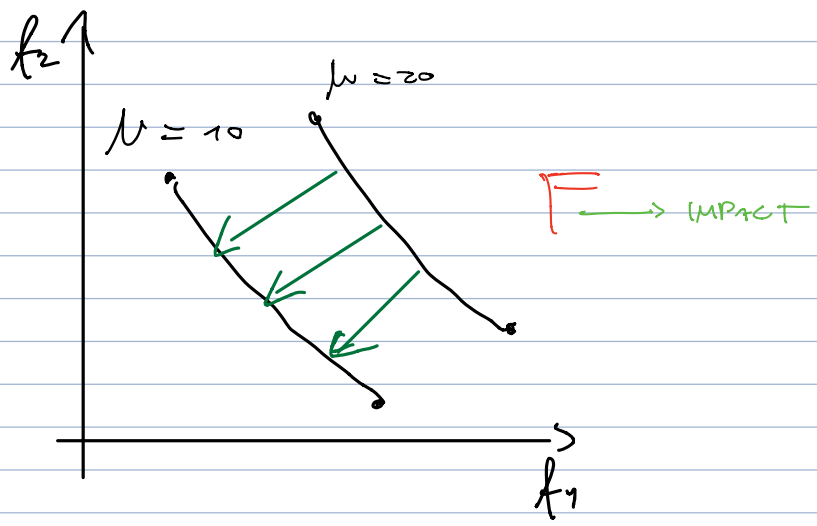
If situation is line



Is this always possible?
Lexicographic order are points. Every point is indifferent to the other but it's strange

2-dim point i have t-1 reciprocally indifference

Reciprocally indifference —> The have exactly the same value of utility



1) they don't cross each other
2) they are ordered

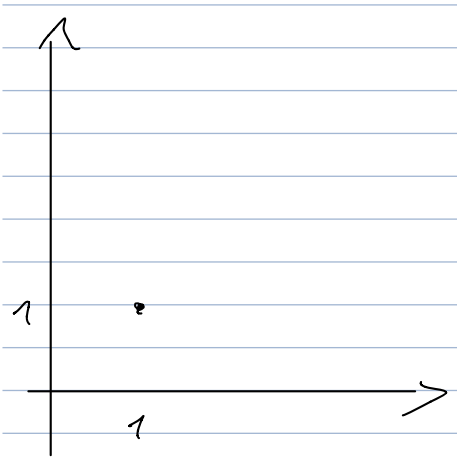
For any two point i can say if they are indifferent curve

Example

Let us assume: $\mu(f) = f_1^2 f_2^3$

$$\max \mu(f(x)) \quad x \in X$$

I Can apply KNN and i can do it point by point



$f_1 = 1/8$
$f_2 = ?$
$\} \in (1,1)$

$\frac{1}{64} f_2^3 = 1 \Rightarrow f_2 = \sqrt[3]{64} = 4$

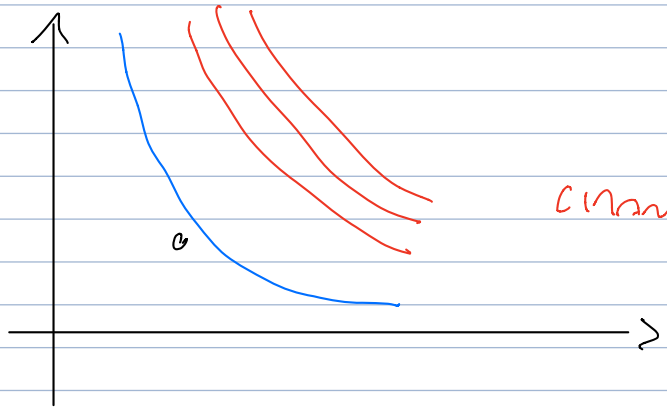
I CAN GENERALIZE THIS!

Assume that utility is a given constant

$$u(x) = c$$

$$x_1^2 x_2^3 = c \qquad x_2^3 = \frac{c}{x_1^2} \qquad x_2 = \sqrt[3]{\frac{c}{x_1^2}}$$

THIS IS SIMILAR TO AN INDIFFERENCE!



CHANGING c

Example

IF I DRAW THEM, THEY ARE THE SAME

$$u(x) = x_1^4 x_2^6 \qquad x_2^6 = \frac{c}{x_1^4} \qquad x_2^6 = \frac{c}{x_1^4} = \sqrt[6]{\frac{c}{x_1^4}} = \sqrt[3]{\frac{\sqrt{c}}{x_1^2}}$$

$$u(x) = 2 \log x_1 + 3 \log x_2$$

IF I DRAW THE CURVE →

SUPPOSE BASE 2

$$\log x_2 = \frac{c - 2 \log x_1}{3}$$

$$x_2 = 2^{\frac{c - 2 \log x_1}{3}}$$

This procedure is reversible and optimum must be the same

But what is useful in practice?

You have preference relation and i want to transform it in a utility function

- I draw in 200-dim and sample impact space is 199 → I DRAW POINTS AND I ASK IF CUSTOMER PREFERS ONE OR ANOTHER

K^p points —> i have to ask which impact he prefers

IMPOSSIBLE! $\frac{K^{\alpha} \cdot K^{\beta}}{2}$

Cob Douglas. ⇒ Utility function can make it easy

$$u(f) = \prod_{e=1}^{n} f_e^{\alpha_e} = A f_1^{\alpha_1} f_2^{\alpha_2} \dots$$

$(1,1) \sim \left(\frac{1}{8}, 4\right)$

If you have some pair I can estimates the exponents

$u(1,1) = u\left(\frac{1}{8}, 4\right)$

$1 = \frac{4^{\alpha_2}}{8^{\alpha_1}} = \frac{2^{2\alpha_2}}{2^{3\alpha_1}} = 2^{2\alpha_2 - 3\alpha_1}$

$0 = 2\alpha_2 - 3\alpha_1$

Summarise

- Example
- Determinate difference curve
- Guess general sample of the family
- Compute coefficients using pair of impacts and normalisation conditions

I don't need to estimate all the values

$$\sum_{e=1}^{e} \alpha_e = 1$$

I'm not only weak-order

Properties ⇒ I want $u(f)$ To be additive

$$u(\bar{f}^1, \bar{f}^2, \bar{f}^3) = \sum_{e=1}^{e} u_e(f_e)$$

$u(f) = f_1^2 f_2^3$

It's not possible to ask 10^{100} impacts to customer, so it's nice to have a function that is additive

$K^2 p$ —> ***Much simpler!***

Also, I can ask not only a person but i can divide the job between different persons.
It's only possible with additive functions

PREFERENCIAL INDEPENDENCE

P = {1, ..., p }
LCP —> only social indicators, are them dependent or independent?

Is this preference the same in environment, finance ecc..?

Unemployment and pollution are always ***BAD.*** ⟶ I ALWAYS WANT THEM LOW

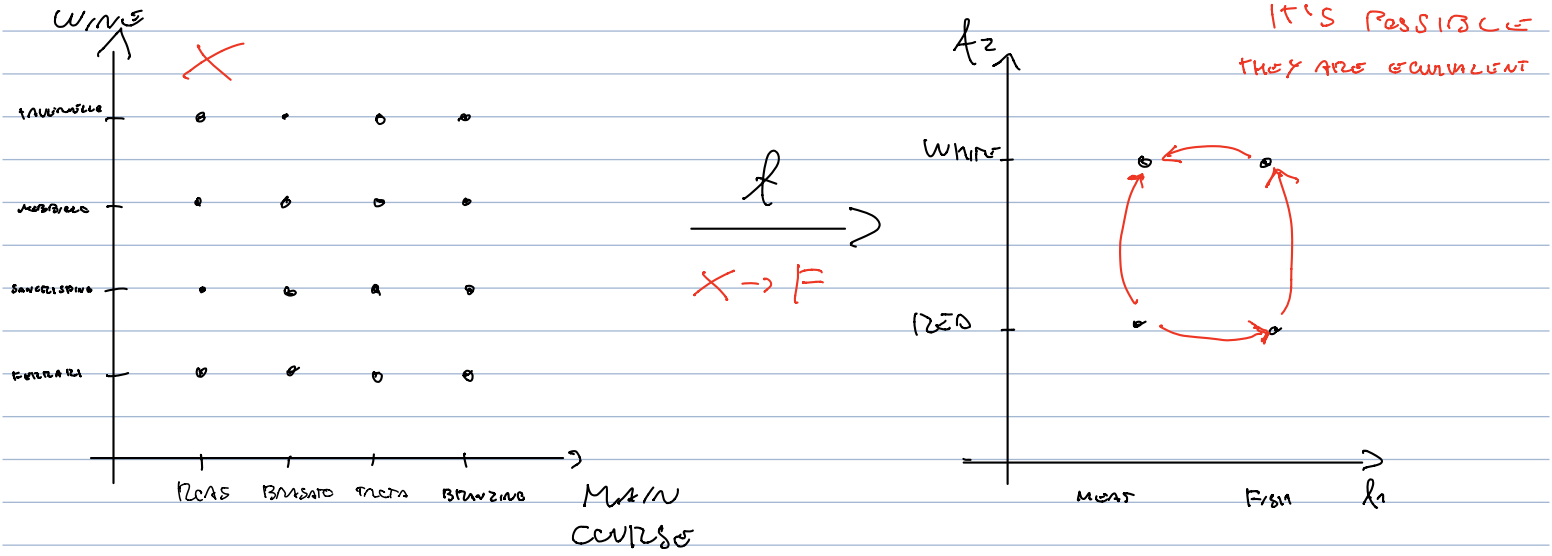
LCP is preference independent from P \ L when

$$\left| \begin{matrix} f_2 \\ \phi \end{matrix} \right| \succsim \left| \begin{matrix} f_2' \\ \phi \end{matrix} \right| \iff \left| \begin{matrix} f_2 \\ \psi \end{matrix} \right| \succsim \left| \begin{matrix} f_2' \\ \psi \end{matrix} \right| \qquad P\{L \mid P\setminus L\}$$

POLLUTION, ACCIDENTS ECC.

$$\forall f_2, f_2', \phi, \psi$$

Example ⟶ DINNER



f_1 = MEAT FISH f_2 = WHITE

IF (M AND M)

RED IS BETTER THAN WHITE

R	$\succ$	W
M		M

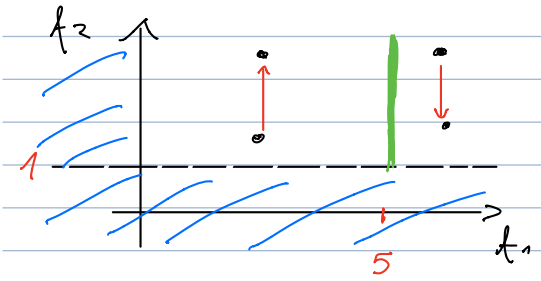
IF (F AND F)

WHITE IS BETTER THAN RED

R	$\not\succ$	W
F		F

I HAVE ALSO TO DO THE OPPOSITE!

$$M(f) = (f_1 - 5) f_2 \qquad F = \{ f \in \mathbb{R} : f_1 \geq 0 \quad f_2 \geq 1 \}$$



I WANT TO MINIMIZE

FIX f_1, IF($f_1 = = 5$) INDIFFERENT CURVE (OR UTILITY)

$f_1 - 5$ IS NEGATIVE

$f_1 = = 6 \qquad \mu = = 2 \longrightarrow$ I WANT LOWER VALUES

f_1 PREFERENCE DEPENDENT OR INDEPENDENT ?

f_2 DEP. TO f_1 $\Rightarrow$ IF f_1 CHANGE: f_2 CHANGE

SO f_2 PREFERENCIALLY DEPENDENT ON f_1

IS f_1 DEPENDENT ?

FIX f_2 WITH VALUE ≥ 1

$\forall$ VALUE OF f_2 I WANT TO GO ON THE RIGHT OF THE GRAPH

Example2

$$\mu(f) = \frac{1}{(f_1 + f_3)(f_2 + f_3)} \qquad F \subseteq \{ f \in \mathbb{R}^3 \}$$

$f_1 \geq 0$

$f_2 \geq 0$

$f_3 \geq 1$

I HAVE TO STAY OVER VALUE 1

THE PROPER SUB SET ARE 6

5 FEATURES $\rightarrow 2^5$ TO CHECK INDEPENDENCE

200 FEATURES $\rightarrow 2^{200}$

SHORT CUT ?

IF INCREASE f_1 ? μ DECREASE

IT'S LIKE A COST

$\Downarrow$

IF POLLUTION INCREASE ? LESS UTILITY

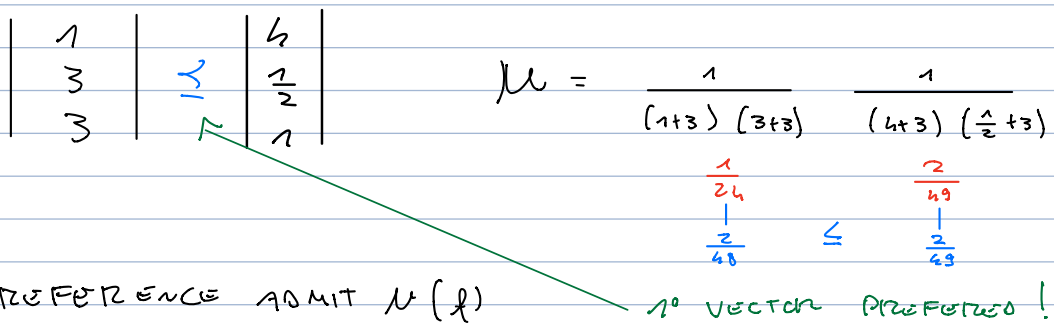
UNFORTUNALLY I CAN'T TRANSFORM THIS IN TO AN ADDITIVE FUNCTION

PAIR (f_1, f_2) IS DEPENDENT ON f_3 ?

L	1	$\geq$	4
P\L	3		$\frac{1}{2}$
	1		1

$$\mu = \frac{1}{(1+1)(3+1)} \qquad \frac{1}{(4+1)\left(\frac{1}{2}+1\right)}$$

$$\frac{1}{8} \qquad \frac{2}{15}$$



ADDITIVE ⟹ PREFERENCIALLY INDEPENDENT

⟸ NOT THIS

Theorem 17: Se una relazione di preferenza ammette una funzione di utillità additiva, allora gode della mutua indipendenza preferenziale.