$(X, \Omega, F, f, D, \pi)$

$X = \{ x \in \mathbb{R}, g_j(x) \leq 0 \; j \leq 1 \}$ WITH
$g_j(x) \in C^2(x)$

$|\Omega| = 1 \iff \Omega = \{\alpha\}$

$F \subseteq \mathbb{R}$

$|D| = 1 \iff D = \{\alpha\}$

π ADMITS A CONSISTENT VALUE FUNCTION
$v : F \to \mathbb{R}$

$\left| \; \text{MIN} \right.$

KNN $\longrightarrow$ WE CAN ENUMERATE THESE POINTS AND FIND OPTIMAL SOLUTION

TH. LOCALLY OPTIMAL POINT?
WHAT IS DIRECTION?

HP. d FEASIBLE DIRECTION

$d \in \mathbb{R} : \exists \; \alpha \; \bar{x}$ AND $\in X \; \forall \alpha \in [0, \alpha]$

$\boxed{\text{TH.}}$ NOT IMPROVING DIRECTION

↳ REAL VECTOR IF EXIST $\overline{\alpha}$
SUCH THAT IF WE START FROM x_2 THIS WILL NOT BE A BETTER POINT THAN $\overline{x}^2$ OBJECTIVE FUNCTION

IT'S INFINITE → IT'S NOT AN ALGORITHM

CRAZIEST ALGORITHM? MOVING ON A CURVE LINE

$S \in \mathbb{R}$
↳ POINT IN n SPACE

$S: \mathbb{R} \to \mathbb{R}^n$

$$\begin{array}{cc} \begin{bmatrix} 2 \sin \alpha \\ 2 \cos \alpha \end{bmatrix} & \begin{array}{c} 0 \\ 2 \end{array} \end{array} \qquad [0, 2]$$

INCREASING α WE ARE MOVING ON THE CIRCLE THEOREM THAT IS FINALLY FEASIBLE

TAYLOR → USE SINGLE VALUE,

MULTIPLY BY α

CHOOSE X

IN WHICH DIRECTION FUNCTION INCREASE?

FEASIBLE SOLUTION!

POINT x^2 IS FEASIBLE AND $g(x) \in C^2(x)$

FEASIBLE REGION

α FEASIBLE ARCH IN X FOR x^2

3 ASSUMPTION FOR NOW

⟶ SUPPOSE g_j

WHERE g GROWS ANGLE > 90°

$$(\nabla g_j(\hat{x}))^T = 0$$

I WANT TO MOVE TO A FEASIBLE REGION

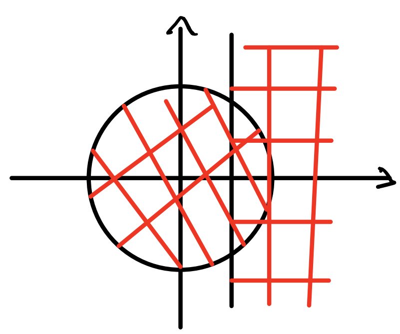
IF WE MAKE SCALAR PRODUCT AND EQUAL TO $\emptyset$

↳ I'M NOT SATISFY MORE AND EITHER LESS

OPT

$$\min f(x) = (x_1 - 1)^2 + x^2$$

$$g_1(x) = -x_1^2 - x_2^2 + 4 \leq 0$$



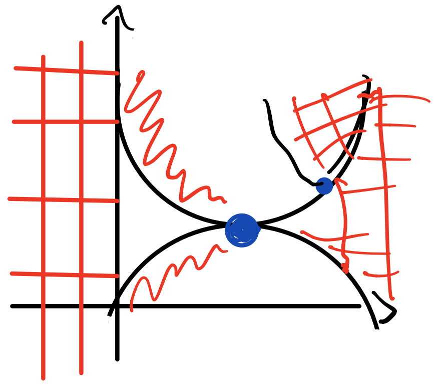
$$g_2(x) = x_1 - \frac{3}{2} \leq 0$$

$$\min \; f(x) = -x_1$$

$$g_1(x) = (x_1 - 1)^3 + (x_2 - 2) \leq 0$$

$$g_2(x) = (x_1 - 1)^3 - (x_2 - 2) \leq 0$$

$$g_3(x) = -x_1 \leq 0$$



$$C = X$$

FOR EACH $x \in C$ DO
  FOR EACH ξ FEASIBLE FOR X IN x
    IF $[\nabla f(x)]^T p_\xi(x) < 0$
      THEN $C = C \setminus \{x\}$

RETURN C

$\boxed{\text{THEOREM}}$

Hp. $\hat{x} \in X$

$g_1 \in C^1(x)$

$\xi(x)$ FEASIBLE ARC FOR X IN $\hat{x}$

Th. $[\nabla g_s(x)]^T p_\xi(\hat{x}) \leq 0 \quad \forall s \in \text{JACK}(\hat{x})$

$\boxed{\text{THEOREM}}$

Hp. $\hat{x} \in X$

$g_1 \in C^1(x)$ $\qquad \nabla g_s \quad s \in \text{JACK}$

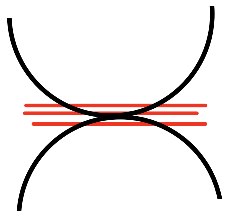
$\hat{x}$ IS A REGULAR POINT $\Rightarrow$ LINEAR INDEPENDENT

$$\left[\nabla g_5(x)\right]^T$$

GRADIANT OF g_5 LINEARLY INDEPENDENT

TH. $\varepsilon(\alpha)$ FEASIBLE FOR X in $\bar{x}$

WE TRYING TO IMAGINE THAT ALL OUR POINTS ARE LINES

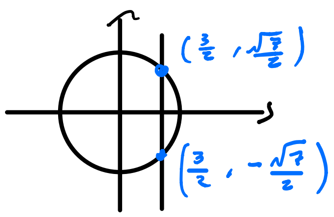


YOU CAN GO LEFT AND ALSO RIGHT

VALUE OF $\frac{3}{2}, \frac{\sqrt{7}}{2}$

$$-\frac{9}{4} - x_2^2 + 4 = 0 \qquad x_2^2 = \frac{7}{4} \Rightarrow x_{1,2} = \pm\frac{\sqrt{7}}{2}$$

WILL SHOW THAT THIS POINTS ARE LINEARLY INDEPENDENT

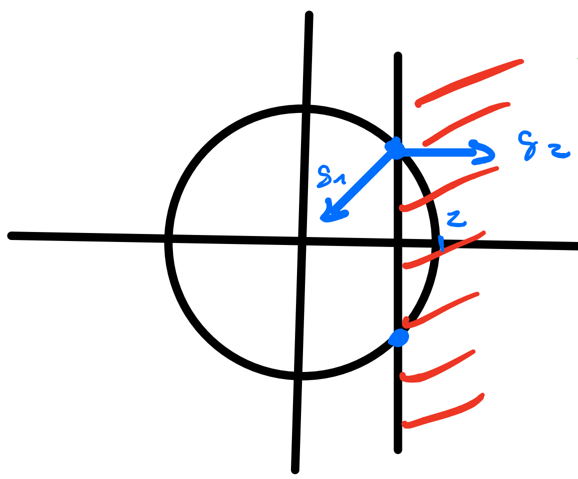


PARTIAL DERIVATIVE

$$\nabla g_1 = \begin{bmatrix} -2x_1 \\ -2x_2 \end{bmatrix} \qquad \nabla g_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\nabla g_1\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right) = \begin{bmatrix} 3 \\ -\sqrt{7} \end{bmatrix}$$

2, ...



CONSTRAIN VIOLETED SO QUICKLY

$$\alpha_1 \nabla g_1 + \alpha_2 \nabla g_2 = 0$$

SAME

FIND LOW REGULAR POINT

$$\alpha_1 \begin{bmatrix} -3 \\ -\sqrt{7} \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$\begin{cases} -3\alpha_1 + \alpha_2 = 0 \\ -\sqrt{7}\,\alpha_1 = 0 \end{cases} \Rightarrow \qquad \alpha_2 = 0 \qquad \alpha_1 = 0$$

THE ONLY POSSIBILITY IS THE GRADIANT $= \emptyset$

g_1 NEVER EQUALS TO $\emptyset$

g_2 NEVER EQUALS TO $\emptyset$

$$\begin{vmatrix} -3 & 1 \\ -\sqrt{7} & 0 \end{vmatrix} \overset{DET}{=} 0+(-\sqrt{7}) = -\sqrt{7}$$

NON $\emptyset$ SO VECTORS ARE INDEPENDENT

MORE WE HAVE THREE GRADIANT

$$\nabla g_1 = \begin{bmatrix} 3(x-1)^2 \\ 1 \end{bmatrix} \qquad \nabla g_2 = \begin{bmatrix} 3(x-1)^2 \\ -1 \end{bmatrix} \qquad \nabla g_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

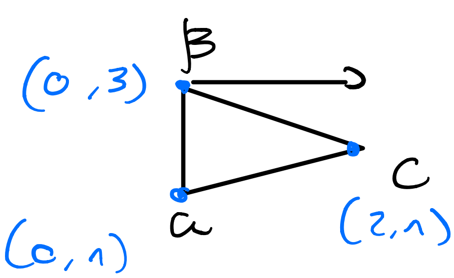
$$\nabla g_1(\beta) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \nabla g_2(a) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

↳ WHICH IS GOOD

$$\nabla g_1(c) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \nabla g_2(c) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

∞ MANY SOLUTIONS

$$\text{DET} \begin{vmatrix} 0 & 0 \\ 1 & -1 \end{vmatrix} = 0 \longrightarrow \text{LINEAR DEPENDENT}$$



ARE REGULAR BUT NOT THE VERTEXES AND WE HAVE TO CHECK THEM

POINT C IS PATOLOGICAL AND NON REGULAR

WHAT WE DO? WE COMPUTE THEM

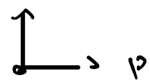
X_{NR} = NONREGULAR POINT X

$C = X \setminus X_{NR}$ $\longrightarrow$ WE ARE CHECKING NON REGULAR POINTS

FOR EACH ...

...

$C = C \cup X_{NR}$

PROBABILITY WITH $-x_1$ WAS 

THIS POINT AS A R MINIMUM

WE CAN SAY TO NOT CHECK FEASIBLE REGION BUT P THAT SATISFY CONDITION OF TANGENT VECTOR OF THE ARCH

X_{NR} = NOT REGULAR(x)

$C = X \setminus X_{NR}$

FOR EACH $x \in C$ do

FOR EACH $p \in \mathbb{R}^n = [\nabla g_j(x)]^T p \leq 0 \quad \forall j \in S_{ACK}$

IF $[\nabla f(x)]^T p < 0$

$C = C \cup X_{NR}$

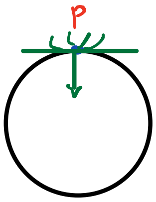
RETURN C

TWO OF LOOPS → NOT NICE

CONSIDER A POINT THAT IS NOT LOCALLY OPTIMAL

$p[\nabla g_j(x)]^T \leq 0 \rightarrow$ ANGLE $> 90°$

WE ARE INTERESTED IN VECTOR p^*

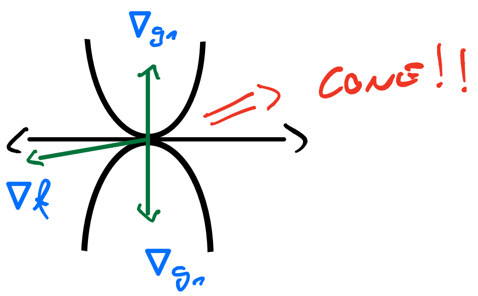


HALF PLANE

WE GOT A CONE! → SET OF VECTORS WITH ANY LENGHT

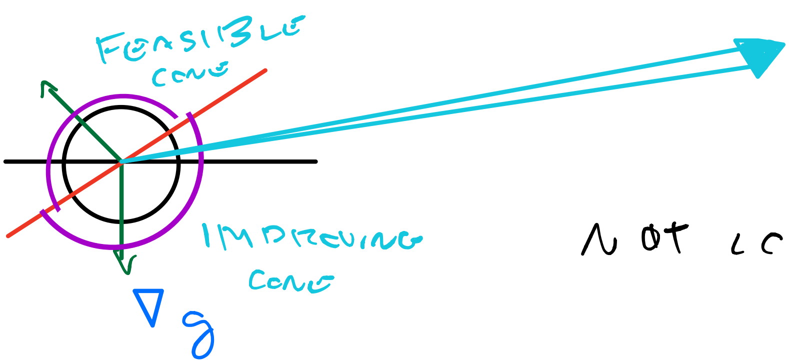
CONE OF VECTOR P THAT SATISFY OUR CONSTRAIN

IT'S KNOWN AS A FEASIBLE CONE



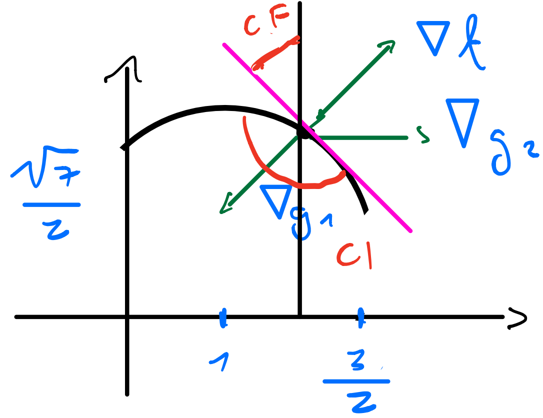
IMPROVING CONE → OBJECTIVE FUNCTION IS GOING TO IMPROVE

$\boxed{\text{IMPROVING CONE AND FEASIBLE CONE NEVER INTERSEPT}}$



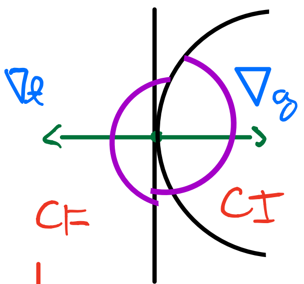
WE CAN MOVE ON THE RIGHT

NOT LOCAL OPTIMUM

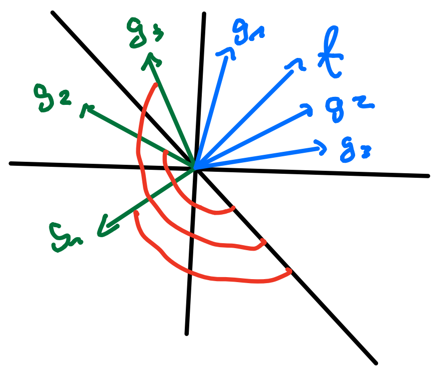


CONE OF CI NOT INTERSECT WITH CONE OF CF

NOT IMPROVING



OPPOSITE GRADE OF ONLY ACTIVE CONSTRAIN



"STAY AWAY FROM g_1, g_2, g_3"

$f \in \mathbb{R}^n$

OTHER VECTORS g_j

$g_j \in \mathbb{R}^n \quad j: 1, \dots m$

INFINITE MANY VECTORS

$p^T f \leq 0$ FOR ALL p: $p^T g_j \leq 0 \quad \forall j = 1 \dots m$

EQUIVALENT

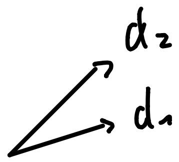
WHAT IS THE RELATION BETWEEN P AND f

$\exists \mu_s \geq 0 : f = \sum_{s=1}^{m} y_s g_s$

COMBINATION & VECTOR
LINEARLY COMB. COEFFICIENT

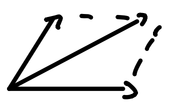
LINEAR COMBINATION $\longrightarrow$ SUM OF NUMBERS MULTIPLY BY VECTOR

IT IS CALLED CONIC COMBINATION



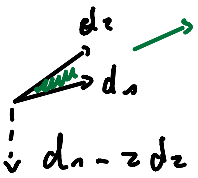
$d_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad d_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

LINEAR COMBINATION OF THE VECTORS ?



SUM PARALLELOGRAMMA DELLE FORZE

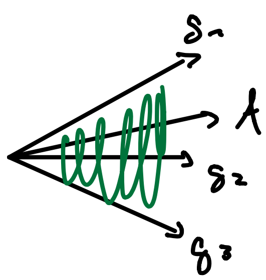
$d_1 - 2d_2 \quad \begin{vmatrix} 0 \\ -3 \end{vmatrix}$



NEVER GET OUT OF THIS CONE IDENTIFY BY THE TWO VECTORS

f IS PART OF THE CONE IDENTIFIED BY g_s VECTOR

I CAN OBTAIN EVERYTHING I WANT



INSTEAD OF f WE CONSIDER $-f$ THEN ALL FALSE

$-P \longrightarrow$ TRASPOSTED P

NOW THIS COMES AS AN ALGORITHM

- $\exists\, \mu_j \geq 0 \quad \nabla f(x) + \sum_{j \in Jack(x)}^{m} \mu_j \nabla g_j = 0$ ➡ X IS A CANDIDATE POINT

- $\mu_j\, g_j(x) = 0$ → IF ACTIVE TRIVIAL, IF NOT ACTIVE IT'S NEGATIVE

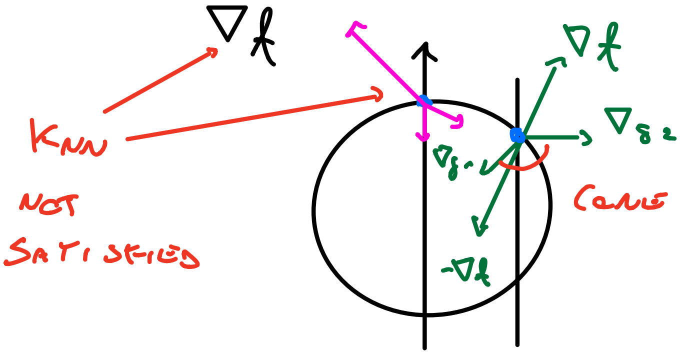
THIS IS A KKN CONDITION

∇g_1 IMPORTANT, ∇g_2 NOT IMPORTANT

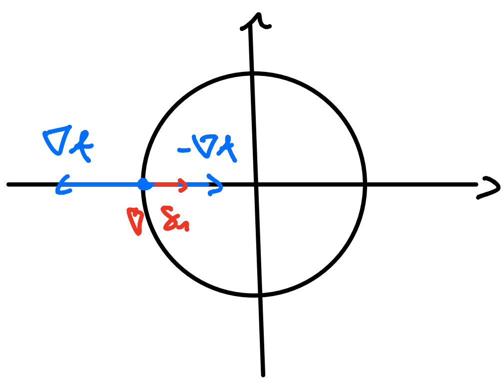
GEOMETRIC POINT OF VIEW?

IT'S ANOTHER CONE → CONE OF THE GRADIENT

$$-\nabla f = \sum \mu_j \nabla g_j$$



$-\nabla f$ INSIDE CONE SO IT'S A CANDIDATE



$-\nabla f$ INSIDE CONE OF GRADIENT AND SATISFY THE CONDITION

$$\min f(x) = (x-1)^2 + x_2$$

$$g_1(x) = -x^2_1 - x_2 + 4 \leq 0$$

$$g_2(x) = x_1 - \frac{3}{2} \leq 0$$

Parz g_2

$$2(x_1 - 1) + \mu_1(-2x_1) + \mu_2(1) = 0$$

$$2x_2 + \mu_1(-2x_2) + \mu_2(0) = 0$$

$$u_1(-x_1^2 - x_2^2 + 4) = 0$$

$$u_2\left(x - \frac{3}{2}\right) = 0$$

$$u_1 \geq 0$$

$$u_2 \geq 0$$

$$x_1 - \frac{3}{2} \leq 0$$

$$-x_1^2 - x_2^2 + 4 \leq 0$$

FIND ALL SOLUTION OF SYSTEMS

4 VARIABLES

N. VARIABLE N. CONSTRAIN

4 CONSTRAIN

N+M VARIABLE = N+M CONSTRAIN

IN KNN