Academic Year 2019-2020

B-74-3-B Time Series Econometrics

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EXERCISE SHEET 1

1.

Let $\{Y\}_{t=-\infty}^{\infty}$ be the stationary and ergodic process generated by the model

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \tag{1}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is independently identically distributed with $E(\varepsilon_t) = 0$, $Var(\varepsilon_t) = \sigma^2$.

(i) Denoting $E((Y_t - \mu)(Y_{t-j} - \mu)) = \gamma_j$, Compute the autocovariances γ_0, γ_1, ...

(ii). Denote $\overline{Y}$ as the sample mean. Using the Beveridge Nelson decomposition, derive the Central Limit Theorem for a suitably standardised version of $\overline{Y}$.

(iii) Introduce the MA(∞) representation $Y_t = \mu + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$. Characterise the parameters ψ_j in terms of the parameters of (1). Compute $\sum_{j=-\infty}^{\infty} \gamma_j$ and verify that

$$\sum_{j=-\infty}^{\infty} \gamma_j = \sigma^2 \left(\sum_{j=0}^{\infty} \psi_j \right)^2$$

2.

Let $\{Y\}_{t=-\infty}^{\infty}$ be a stationary and ergodic process with $E(Y_t) = \mu$, $E((Y_t - \mu)(Y_{t-j} - \mu)) = \gamma_j$ and denote $\overline{Y}$ as the sample mean and $\widehat{\gamma}_j$ as the sample autocovariance. Suppose that we have a sample of 100 observations (i.e., $T = 100$), and we estimated $\overline{Y} = 4$, $\widehat{\gamma}_0 = 2$, $\widehat{\gamma}_1 = 1$, $\widehat{\gamma}_2 = 0.25$, $\widehat{\gamma}_3 = -0.25$, and that we want to test $H_0 : \{\mu = 3\}$ against the alternative $H_0 : \{\mu \neq 3\}$ at 5% significance level.
Describe the test statistic you are going to use and its limit distribution under the null, and then perform the test.

3.

Are the stochastic processes generated by the following models stationary?

i. $Y_t = \varepsilon_t + 1.6\varepsilon_{t-1} + 0.48\varepsilon_{t-2}$, $\varepsilon_t \sim wn\,(0, \sigma^2)$;

ii. $Y_t = Y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim wn\,(0, \sigma^2)$ for $t > 0$, $Y_0 = 0$;

iii. $Y_t = \varepsilon_t \varepsilon_{t-1}$, $\varepsilon_t \sim NID\,(0, \sigma^2)$.

iv. $Y_t = 0.8Y_{t-1} - 0.8Y_{t-2} + \varepsilon_t$, $\varepsilon_t \sim wn\,(0, \sigma^2)$;

v. $Y_t = N\,(1, 1)$ for t odd, Y_t exponentially distributed with mean 1 for t even, Y_t independent over time.

vi. $Y_t = \varepsilon_t$ and ε_t is a sequence of independent identically distributed Cauchy random variables.

4.

i. Let $\{Z\}_{t=-\infty}^{\infty}$ be a stationary ARMA process. What does it mean to say that $\{Z\}_{t=-\infty}^{\infty}$ is invertible?

ii. Consider the process $\{Y\}_{t=-\infty}^{\infty}$ generated by the model

$$Y_t = 2 + \varepsilon_t + 1.6\varepsilon_{t-1} + 0.48\varepsilon_{t-2}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is white noise $(0, \sigma^2)$ (i.e., ε_t is $\mathrm{wn}(0, \sigma^2)$).

Is the process $\{Y\}_{t=-\infty}^{\infty}$ invertible?

iii. Derive the autocorrelation function of $\{Y\}_{t=-\infty}^{\infty}$ in part (ii) up to $j = 5$.

5.

Consider the process $\{Y\}_{t=-\infty}^{\infty}$ generated by the model

$$Y_t = \varepsilon_t + 0.5\varepsilon_{t-1}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is white noise $(0, \sigma^2)$ (i.e., ε_t is $\mathrm{wn}(0, \sigma^2)$).

i. Compute the best linear forecast of Y_{t+1} when it is known that $Y_t = 0.8$

ii. Compute the best linear forecast of Y_{t+1} when it is known that $Y_t = 0.8$, $Y_{t-1} = 1.2$

iii. Compute the partial autocorrelation function of $\{Y\}_{t=-\infty}^{\infty}$ up to $j = 3$.