

Optimization (Decision methods and models)

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16th December 2019

Available time: 2 hours

Note: explicitate all assumptions and motivate the computational steps.

- Exercise 1** - Briefly define the concepts of *impact* and *indicator* in a decision problem, explaining their role in the decision process.
- a) What is the meaning of the expression *money pump*? What kind of preference relation makes it possible?
- b) Briefly define the concept of a *consistent value function* for a preference relation.

Exercise 2 - Given the following Mathematical Programming problem:

$$\begin{aligned} \min f(x) &= -x_1 - x_2 \\ g_1(x) &= x_1^2 + x_2 - 2 \leq 0 \\ g_2(x) &= x_1^2 - x_2 \leq 0 \\ g_3(x) &= -x_1 \leq 0 \end{aligned}$$

- a) represent the problem graphically;
- b) determine the nonregular points or show that none exists;
- c) determine the candidate point according to Karush-Kuhn-Tucker conditions, and in particular the global minimum point(s).

Exercise 3 - With respect to the multiple-attribute decision problems:

- a) define the concept of Paretian preference;
- b) list the methods discussed in the course to find the Paretian region;
- c) given the following problem with five alternatives and two indicators:

	a_1	a_2	a_3	a_4	a_5
f_1	3	1	0	6	2
f_2	4	5	9	1	2

find the support of each alternative performing a sensitivity analysis on the weight w of indicator f_1

Exercise 4 - Briefly describe the following concepts of the *Analytic Hierarchy Process*:

- a) the use of pairwise comparison: how does it work and what is its aim?
- b) the use of qualitative scales: how does it work and what is its aim?
- c) the problem of *rank reversal*: what is it and how can it be handled?

Exercise 5 - Given the following problem in conditions of ignorance, in which the reported values represent **benefits**:

$u_{\omega a}$	a_1	a_2	a_3	a_4	a_5
ω_1	30	60	10	20	50
ω_2	50	40	80	30	50
ω_3	70	20	80	60	50

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So

- a) solve it with the *worst-case criterium*;
- b) solve it with the *Laplace criterium*;
- c) solve it with the *regret criterium*.

Exercise 6 - With respect to the problems in conditions of risk:

- a) briefly discuss the formal defects of the expected value criterium;
- b) define the concept of *lottery* according to Von Neumann and Morgenstern;
- c) solve the following problem with the expected value criterium and solve it again with the stochastic utility theory, assuming a utility function $u(f) = \sqrt{f}/10$ in the impact set $F = [0, 10]$:

$f(x, \omega)$	ω_1	ω_2	ω_3	ω_4
x_1	100	9	4	64
x_2	49	9	36	49

$\pi(\omega)$	0.1	0.3	0.4	0.2
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$a^2 b^2 \neq 1$
 2
 $\sqrt{3} -$
 4
 $a^2 b^2 \neq 5 \sim$
 $a^2 b^2 \neq 6 -$