Academic Year 2019-2020

B-74-3-B Time Series Econometrics

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EXERCISE SHEET 4

1.

Assume that

$$\begin{aligned} Y_t &= Y_{t-1} + u_t \text{ when } t > 0 \\ Y_0 &= 0 \end{aligned}$$

and that we observed $Y_t = 0.2$, $Y_{t-1} = 0.4$, $Y_{t-2} = 0.2$, $Y_{t-3} = -0.1$.

Compute the best linear forecast of Y_{t+1} if u_t is a AR(2) with

$$u_t = 0.4u_{t-1} + 0.2u_{t-2} + \varepsilon_t, \text{ where } \varepsilon_t \ iid\left(0, \sigma^2\right).$$

2.

Let $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ be $iid\,(0, \sigma^2)$ and assume that

$$\begin{aligned} Y_t &= Y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} \text{ when } t > 0 \\ Y_0 &= 0 \end{aligned}$$

and that we have a time series $Y_1, ..., Y_T$.

i. Explain (very briefly) how you would estimate θ in this case.

ii. Suppose that we estimated $\widehat{\theta} = 0.8$ by Conditional Maximum Likelihood, and we computed the residuals $\widehat{\varepsilon}_2, ..., \widehat{\varepsilon}_T$. Assuming that $Y_T = 0.2$, $Y_{T-1} = -0.1$, $\widehat{\varepsilon}_T = 0.1$, compute an approximation to the best linear forecast.

3.

Let ε_t be white noise with $E\,(\varepsilon_t) = 0$, $E\,(\varepsilon_t^2) = \sigma^2$, and

$$\begin{aligned} Y_t &= c + Y_{t-1} + \varepsilon_t \text{ if } t > 0, \\ Y_t &= 0 \text{ if } t \leq 0. \end{aligned}$$

Show that if $c \neq 0$ then Y_t has a linear trend (hint: using repeated substitutions, rewrite $Y_t = ct + \sum_{j=1}^{t} \varepsilon_j$)

4.

What do you mean by saying that:

i. a certain process $\{X_t\}_{t=-\infty}^{\infty}$ is integrated of order 1.

ii. a certain process $\{Z_t\}_{t=-\infty}^{\infty}$ is integrated of order 2.

iii. How can you use the Dickey and Fuller / Augmented Dickey and Fuller test to establish the order of integration of a process?

5.Let $\{Y_t\}_{t=-\infty}^{\infty}$ be the process generated by the model

$$Y_t = \alpha + \phi Y_{t-1} + \varepsilon_t$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent, identically distributed process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$, and α is a constant such that $\alpha = 0$ if $\phi = 1$. To check if Y_t has a unit root, we take a sample Y_1, ..., Y_{100} (i.e., $T = 100$), and we run the regression

$$Y_t = \underset{(0.17)}{0.54} + \underset{(0.07)}{0.75} Y_{t-1} + \widehat{\varepsilon}_t$$

(estimated standard errors in parenthesis).

(i) Test $H_0 : \{\phi = 1\}$ with 5% significance level.

(ii) Consider an alternative test, where we estimate

$$Y_t = \underset{(0.05)}{0.89} Y_{t-1} + \widehat{\varepsilon}_t$$

and test for a unit root in this model. Discuss advantages and disadvantages of this procedure, and explain which test you would recommend (or, if you recommend running both tests, explain why you would recommend to do that).

(iii) Let $\{X_t\}_{t=-\infty}^{\infty}$ be the process generated by the model

$$X_t = \beta + \rho X_{t-1} + u_t$$

where $\{u_t\}_{t=-\infty}^{\infty}$ is a stationary AR(1) process $u_t = \phi u_{t-1} + \varepsilon_t$ and $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is a independent, identically distributed process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$, and β is a constant such that $\beta = 0$ if $\rho = 1$.

Suppose we are interested in testing $H_0 : \{\rho = 1\}$ and we estimate the regression

$$\Delta X_t = \underset{(0.13)}{0.13} - \underset{(0.03)}{0.08} X_{t-1} + \underset{(0.09)}{0.44} \Delta X_{t-1} + \widehat{\varepsilon}_t$$

Test $H_0 : \{\rho = 1\}$.