UNIVERSITÀ DEGLI STUDI DI MILANO
Dipartimento di Economia, Management
e Metodi Quantitativi

B-74-3-B Time Series Econometics

Specimen Paper #2

Time allowed: 90 Minutes

Authorized material:

- Non-programmable calculator (for personal use only)
- One A4 page (one-sided) of handwritten, personal notes (for personal use only)

Material provided at the time of the exam:

- Probability Tables as on ARIEL

During the exam:

- Put your student card in a visible place to facilitate identity control
- No questions will be answered
- You are not allowed to leave the room

The Exam is divided in TWO parts: Questions 1 to 4 are Short Questions; Questions 5 and 6 are Long Questions. Answers to Short Questions are worth 12.5% of the final mark per question; Answers to Long Questions are worth 25% of the final mark per question.

Full marks may be obtained by complete answers to ALL six questions.

A complete answer to a question should include a clear statement of all the necessary steps in the argument, together with any assumptions and working.

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Question 1 (12.5% of total mark).

Let $\{Y_t\}_{t=-\infty}^{\infty}$ be the process generated by the AR(1) model

$$Y_t = \phi Y_{t-1} + \varepsilon_t$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Show that when $|\phi| < 1$, the process is stationary.

Question 2 (12.5% of total mark).

Let $\{Y_t\}_{t=-\infty}^{\infty}$ be the process generated by the invertible MA(1) model

$$Y_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Suppose that we have a time series $\{Y_1, \dots, Y_{99}\}'$ (i.e., T = 99) and that we estimated, by maximum likelihood, two models: an MA(1), and an MA(2),

$$Y_t = \varepsilon_t + \theta\varepsilon_{t-1},\ \text{MA(1)}$$

$$Y_t = \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2},\ \text{MA(2)}$$

and that we test $H_0\colon\{\theta = 0.9\}$ in the MA(1) model and $H_0\colon\{\theta_1 = 0.9,\ \theta_2 = 0\}$ in the MA(2) model.
Outputs of the estimation and Wald test for the two models are displayed in the next two pages.

(Question 2 continues on the next page)

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(Question 2, continued)

Estimation and test in the MA(1) model

Dependent Variable: Y
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 1 99
Included observations: 99
Convergence achieved after 33 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.068594	0.157992	0.434161	0.6651
MA(1)	0.730583	0.084889	8.606327	0.0000
SIGMASQ	0.833208	0.096710	8.615568	0.0000

R-squared	0.390882	Mean dependent var	0.074318
Adjusted R-squared	0.378192	S.D. dependent var	1.175521
S.E. of regression	0.926955	Akaike info criterion	2.723719
Sum squared resid	82.48761	Schwarz criterion	2.802359
Log likelihood	-131.8241	Hannan-Quinn criter.	2.755537
F-statistic	30.80244	Durbin-Watson stat	1.840167
Prob(F-statistic)	0.000000		

Inverted MA Roots	-.73

Wald Test:
Equation: Untitled

Test Statistic	Value	Df	Probability
t-statistic	8.606327	96	0.0000
F-statistic	74.06886	(1, 96)	0.0000
Chi-square	74.06886	1	0.0000

Null Hypothesis: C(2)=0.
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(2)	0.730583	0.084889

Restrictions are linear in coefficients.

(Question 2 continues on the next page)

(Question 2, continued)

Estimation and test in the MA(2) model

Dependent Variable: Y
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 1 99
Included observations: 99
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.066712	0.172953	0.385720	0.7006
MA(1)	0.799120	0.121864	6.557489	0.0000
MA(2)	0.089314	0.108816	0.820784	0.4138
SIGMASQ	0.826064	0.113130	7.301906	0.0000

R-squared	0.396105	Mean dependent var	0.074318
Adjusted R-squared	0.377034	S.D. dependent var	1.175521
S.E. of regression	0.927818	Akaike info criterion	2.735568
Sum squared resid	81.78034	Schwarz criterion	2.840421
Log likelihood	-131.4106	Hannan-Quinn criter.	2.777991
F-statistic	20.77066	Durbin-Watson stat	1.979879
Prob(F-statistic)	0.000000		

Inverted MA Roots	-.13	-.66

Wald Test:
Equation: Untitled

Test Statistic	Value	Df	Probability
F-statistic	2.693180	(2, 95)	0.0728
Chi-square	5.386361	2	0.0677

Null Hypothesis: C(2)=0.9, C(3)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-0.9 + C(2)	-0.100880	0.121864
C(3)	0.089314	0.108816

Restrictions are linear in coefficients.

Compare the outcomes for the two models and explain what may have caused any relevant difference that you observed.

Question 3 (12.5% of total mark).

Let $\{Y_t\}_{t=-\infty}^{\infty}$ be the process generated by the AR(2) model

$$Y_t \ = 1.2Y_{t-1} - 0.72Y_{t-1} + \varepsilon_t$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E\ (\varepsilon_t\ )\ =\ 0$, $E\ (\varepsilon_t^2\ )\ =\ \sigma^2$.

3.1) Check if the process $\{Y_t\}_{t=-\infty}^{\infty}$ is stationary.

3.2) Define the Impulse Response Function for stationary ARMA(p,q) processes.

3.3) The Impulse Response Function (IRF) for the lags 1 to 10 takes values

Lags	1	2	3	4	5	6	7	8	9	10	11	12
AC	0.70	0.12	-0.36	-0.52	-0.36	-0.06	0.19	0.27	0.19	0.03	-0.10	-0.14

Comment on the pattern of the Impulse Response Function.

Question 4 (12.5% of total mark).

What does it mean that we should follow “parsimonious modelling” when selected a model for a process?

Question 5 (25% of total mark).

5.1) What does it mean to say that a process is I(0)?
What does it mean to say that a process is I(1)?

5.2)
Consider processes $\{Y_t\}_{t=-\infty}^{\infty}$ and $\{Z_t\}_{t=-\infty}^{\infty}$ defined as

$$Y_t = Y_{t-1} + v_t$$

$$Z_t = Z_{t-1} + u_t$$

where $\{v_t\}_{t=-\infty}^{\infty}$ and $\{u_t\}_{t=-\infty}^{\infty}$ are I(0) processes and v_t is independent of u_s for all t,s when $t > 0$, and $Y_t = 0$, $Z_t = 0$ when $t \leq 0$.
Suppose that we estimated the regression

Dependent Variable: Y
Method: Least Squares
Sample: 1 500
Included observations: 500

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.054899	0.414749	17.01003	0.0000
Z	-0.113204	0.010822	-10.46024	0.0000

R-squared	0.180134	Mean dependent var	5.460262
Adjusted R-squared	0.178488	S.D. dependent var	9.515807
S.E. of regression	8.624870	Akaike info criterion	7.151169
Sum squared resid	37045.42	Schwarz criterion	7.168027
Log likelihood	-1785.792	Hannan-Quinn criter.	7.157784
F-statistic	109.4166	Durbin-Watson stat	0.086013
Prob(F-statistic)	0.000000		

Comment on this regression output. What does it say that this is a “spurious regression”?

5.3) How would you model the relation between processes $\{Y_t\}_{t=-\infty}^{\infty}$ and $\{Z_t\}_{t=-\infty}^{\infty}$?

Question 6 (25% of total mark).
Consider the bivariate process generated by the VAR(2) model

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} \phi_5 & \phi_6 \\ \phi_7 & \phi_8 \end{bmatrix} \begin{bmatrix} X_{t-2} \\ Y_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ e_t \end{bmatrix} \qquad \text{VAR(2)}$$

where $u_t = (\epsilon_t, v_t)'$ is an independent process with $E(u_t u_t') = \Sigma$.

6.1) What restriction on the coefficients would you test, to check if Y_t does not Granger causes X_t?

6.2) What does it mean that Granger causality is not causality? As part of your answer, provide a realistic example of a situation in which Granger causality does not imply causality.

6.3) Introduce the Structuralised IRF for this VAR(2) and explain why the definition of the orthogonalized innovations poses an identification problem.