# Prerequisites for the course on "Decision methods and models"

The following concepts will be given for granted and only briefly recalled during the course. Students unfamiliar with these concepts should ponder whether they feel really able to catch up.[1]

## Mathematical analysis

### Gradient vector of a function

$$f(x) = (x_1 - 1)^2 + x_2^2 \Rightarrow \nabla f(x) = \begin{bmatrix} 2(x_1 - 1) \\ 2x_2 \end{bmatrix}$$

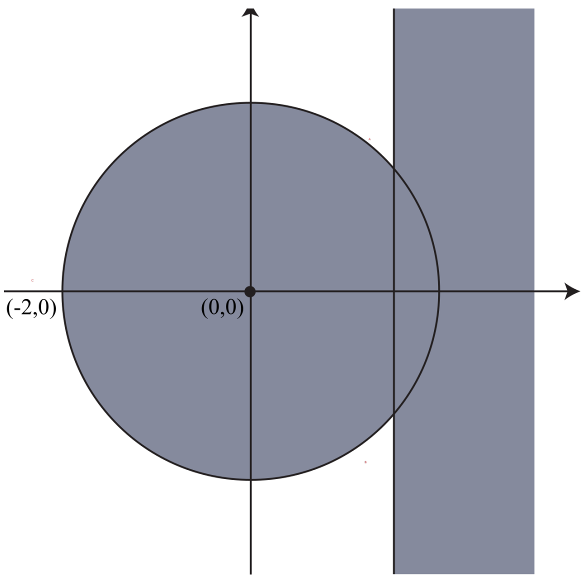
$$g(x) = x_1 - 3/2 \Rightarrow \nabla g(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

### Scalar product of vectors

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \; d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \Rightarrow d_1 \cdot d_2 = 3 \cdot 1 + 2 \cdot (-4) = -5$$

### Graphical representation of constraints

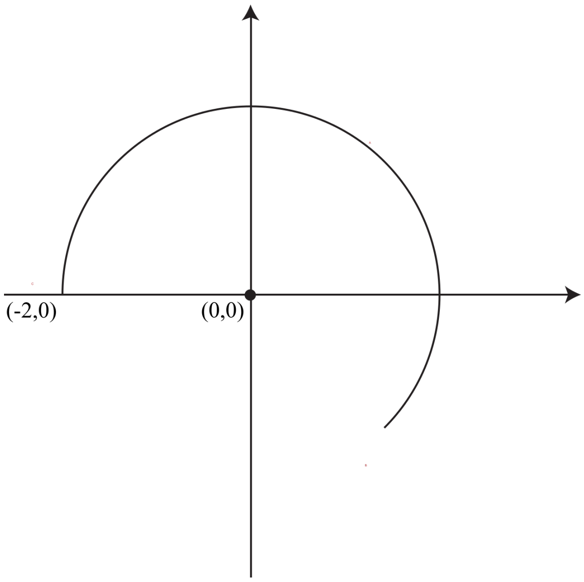
$$\begin{aligned} g_1(x) &= -x_1^2 - x_2^2 + 4 \leq 0 \\ g_2(x) &= x_1 - 3/2 \leq 0 \end{aligned}$$




[1] This document has been written very quickly, and will be updated as soon as possible. Thank you in advance for pointing out mistakes in the examples or missing prerequisites.

### Lines in parametric form

$$\begin{cases} \xi_1(\alpha) = -2\cos(\alpha) \\ \xi_2(\alpha) = 2\sin(\alpha) \end{cases} \quad \text{for } \alpha \in \left[0, \frac{5}{4}\pi\right]$$




# Linear algebra

### Linearly (in)dependent vectors

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\alpha_1 d_1 + \alpha_2 d_2 = 0 \Rightarrow \begin{cases} 3\alpha_1 + \alpha_2 = 0 \\ 2\alpha_1 - 4\alpha_2 = 0 \end{cases} \Rightarrow \alpha_1 = \alpha_2 = 0 \Rightarrow \text{Independent vectors}$$

$$d_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \text{ and } d_2 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix}$$

$$\alpha_1 d_1 + \alpha_2 d_2 = 0 \Rightarrow \begin{cases} 3\alpha_1 + \alpha_2 = 0 \\ 2\alpha_1 + \frac{2}{3}\alpha_2 = 0 \end{cases} \Rightarrow \alpha_2 = -3\alpha_1 \Rightarrow \text{Dependent vectors}$$

### Determinant of a matrix

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow |D_1| = 3 \cdot (-4) - 2 \cdot 1 = -14$$

$$D_2 = \begin{bmatrix} 3 & 1 \\ 2 & 2/3 \end{bmatrix} \Rightarrow |D_2| = 3 \cdot \frac{2}{3} - 2 \cdot 1 = 0$$

### Eigenvalues and eigenvectors

$$D_1 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} \Rightarrow |\lambda I - D_1| = (\lambda - 3) \cdot (\lambda + 4) - (-2) \cdot (-1) = \lambda^2 + \lambda - 14 = 0 \text{ etc.}$$

## Probability calculus

$$\pi\,(A) = 0.4 \quad \pi\,(B) = 0.3 \quad \pi\,(AB) = 0.2 \Rightarrow \pi\,(A + B) = 0.5$$

$$\pi\,(A) = 0.4 \quad \pi\,(B|A) = 0.2 \Rightarrow \pi\,(AB) = 0.4 \cdot 0.2 = 0.08$$

$$\begin{cases} |\Omega| = 4 \\ \pi\,(\omega) = [\ 0.1\ 0.3\ 0.4\ 0.2] \\ f\,(\omega) = [\ 10\ 4\ 6\ 2] \end{cases} \quad \Rightarrow E\,[f] = 10 \cdot 0.1 + 4 \cdot 0.3 + 6 \cdot 0.4 + 2 \cdot 0.2 = 5$$