Università degli Studi di Milano   Corso di studi in Informatica/Matematica

Decision methods and models
(Prof. Roberto Cordone)
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Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Briefly define the concepts of *solution* and *decision variable* in a decision problem, explaining their role in the decision process.

What does it mean that two impacts are indifferent? And incomparable?

Given a decision problem with impact set $F = \{a, b, c, d, e\}$ and preference relation $\Pi = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, d), (c, a), (c, b), (c, c), (c, d), (d, d), (e, a), (e, b), (e, c), (e, d), (e, e)\}$:

a) list the main properties enjoyed by Π, deducing whether it is some kind of order;

b) derive the associated indifference relation Ind_Π;

c) derive the associated strict preference relation Str_Π.

Exercise 2 - Given the following mathematical programming problem

$$\min f(x) = -x_1 - x_2$$
$$g_1(x) = -x_1 - x_2 \leq 0$$
$$g_2(x) = x_1 - 2 \leq 0$$
$$g_3(x) = x_1 + x_2^2 - 2 \leq 0$$

a) give a graphical representation of the problem;

b) find the *nonregular points* (if any exist);

c) determine the candidate points according to the *Karush-Kuhn-Tucker conditions*, and in particular the global minimum points.

Exercise 3 - Briefly define the concept of *indifference curve*.

Briefly define the concept of *utility function*.

The following table evaluates 5 solutions with 2 **cost** indicators.

	a_1	a_2	a_3	a_4	a_5
f_1	0	14	5	31	20
f_2	82	82	37	42	6

Enumerate the Paretian solutions applying the definition.

Choose a solution with the *utopia point* method using the Manhattan distance L_1.

Exercise 4 - List the main properties required from a *pairwise comparison matrix* to be *consistent*.

Briefly describe the *Eigenvector method* to obtain a weight vector from an inconsistent pairwise comparison matrix.

Briefly describe a method to turn the outranking relation of the *ELECTRE methods* into a weak order.

Given the impact set $F = \{a, b, c, d, e\}$ and the outranking relation $S = \{(a, a), (a, b), (b, b), (b, d), (c, c), (c, e), (d, d), (e, a), (e, b), (e, d), (e, e)\}$, determine the *kernel*.

Exercise 5 -   Given the following decision problem in conditions of uncertainty, in which the values indicate **benefits**:

$u_{a\omega}$	ω_1	ω_2	ω_3	ω_4
a_1	85	60	0	65
a_2	90	50	40	30
a_3	45	85	45	100

a) explain the meaning of the symbols a, ω, $u_{a\omega}$;

b) apply *Hurwicz criterium* parametrically, indicating the alternative chosen for different values of the *pessimism coefficient* ρ;

c) apply the *regret criterium* to solve the problem.

Exercise 6 -   Describe the *expected value criterium* and its formal defects.

Define the concept of *lottery* according to Von Neumann and Morgenstern.

Briefly define the *continuity axiom* of Von Neumann and Morgenstern's theory.

What is the *risk profile* of a decision-maker and what information does it provide?

Exercise 7 -   Define the concept of *Nash equilibrium* in a two-player game.

Briefly define the concept of *pure coordination game*.

In the following two-player game, the values indicate *payoffs*:

	a	b	c
a	(19,9)	(23,23)	(13,16)
b	(7,12)	(6,16)	(5,6)
c	(6,6)	(5,9)	(27,12)

Determine the dominated strategies, if any exist.

Determine the Nash equilibria, if any exist.

Exercise 8 -   Briefly define the concept of *minimal decisive set* for a pair of impacts according to Arrow's theory.

A group D of decision-makers has a set $X = \{a, b, c\}$ of alternatives. Their preferences are described in the following table.

Preferenza	Number of decision-makers
$a \prec b \prec c$	5
$b \prec a \prec c$	4
$b \prec c \prec a$	10
$c \prec b \prec a$	7
$a \prec b \approx c$	10

Which alternative is chosen applying the *Condorcet method*?

Which alternative is chosen applying the *Borda method*?

Which alternative is chosen applying the *plurality system*?