Università degli Studi di Milano Corso di studi in Informatica/Matematica

Decision methods and models
(Prof. Roberto Cordone)
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Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Briefly describe the difference between *prescriptive* and *descriptive* models and their roles in a decision process.

Briefly define the concepts of *solution* and *decision variable* in a decision problem.

Briefly define the concept of *stakeholder* in a decision problem.

Briefly describe the main similarity and the main difference between game theory and group decision problems.

Exercise 2 - Given the following mathematical programming problem

$$\begin{aligned} \min f(x) &= -x_1 + x_2 \\ g_1(x) &= x_1^2 + 4x_2^2 - 4 \leq 0 \\ g_2(x) &= x_1 - x_2 - 2 \leq 0 \\ g_3(x) &= -x_1 \leq 0 \end{aligned}$$

a) give a graphical representation of the problem;

b) find the *nonregular points* (if any exist);

c) determine the candidate points according to the *Karush-Kuhn-Tucker conditions*, and in particular the global minimum points.

Exercise 3 - Referring to multiple attribute utility theory, define the concept of *indifference curve*.

Determine the utility function $u(f)$ of a problem with three indicators, assuming it to belong to the Cobb-Douglas family ($u(f) = f_1^{\alpha_1} f_2^{\alpha_2} f_3^{\alpha_3}$), and the impacts $A = (1, 1, 1)$, $B = (2, 1/2, 1)$ and $C = (1, 1/8, 16)$ to be indifferent.

Is $u(f)$ additive? Are the attributes mutually preferentially independent?

Briefly describe the *weighted-sum method* to determine the Paretian region of a multiple objective decision problem.

Exercise 4 - Considering the *ELECTRE* methods:

a) briefly describe their basic criticism to the classical multi-attribute utility theory;

b) given the following evaluation matrix, whose values represent **benefits**

u_{fa}	a_1	a_2	a_3	a_4	a_5
f_1	0.80	0.30	0.76	0.32	0.26
f_2	0.20	0.90	0.30	0.85	0.96

build the *outranking relation* based on thresholds $\epsilon_1 = \epsilon_2 = 0.05$ and the resulting *kernel*, motivating the process;

c) build the *concordance* matrix C restricted to the alternatives of the kernel, with the weight vector $w = [\, 0.6\ 0.4 \,]'$, and select the best one according to the *concordance index* $C_f = \sum_{f' \in K} (c_{ff'} - c_{f'f})$.

Exercise 5 - Briefly describe the difference among the concepts of *uncertainty*, *ignorance* and *risk*.

Briefly describe the *worst-case criterium* and explain whether introducing additional alternatives could affect the order of the other ones.

Briefly describe the *Laplace criterium* and explain whether it always imposes a weak order on the alternatives.

Briefly describe the *surplus criterium* and explain whether duplicating a scenario could affect the order of the alternatives.

Exercise 6 - A decision-maker wants to buy a second-hand car. A local dealer sells one for 1200 Euros, with a warranty: any faults appearing in the first 3 months will be fixed for free. A private owner sells the same model, in apparently similar conditions, for 800 Euros with no warranty: the probability of a fault occurring in the first 3 months is 0.5, and the repairment cost is 600 Euros. The decision-maker also found a mechanic who could check the car. If a fault is present, the mechanic could fail to find it, with a probability of 0.2; if there is no fault, the mechanic will certainly find none.

Model the problem with a decision tree.

What is the maximum sum for which it is still reasonable to hire the mechanic?

Exercise 7 - Determine the equilibrium points (if any) for the following two-player game, where the *payoffs* represent **benefits**:

	a	b	c
a	(4,2)	(8,5)	(0,3)
b	(2,6)	(9,1)	(6,0)
c	(1,9)	(1,3)	(7,5)

Determine the *value* of the previous game for the two players.

Give the appropriate names to the following two-player symmetric games.

G_1

	a	b
a	(6,6)	(1,3)
b	(3,1)	(0,0)

G_2

	a	b
a	(5,5)	(0,8)
b	(8,0)	(2,2)

G_3

	a	b
a	(5,5)	(0,4)
b	(4,0)	(3,3)

G_4

	a	b
a	(0,0)	(6,5)
b	(5,6)	(8,8)

Exercise 8 - Briefly define the concept of *social welfare function* as a method to aggregate preferences.

Briefly describe the *lexicographic method* to obtain a group preference relation.

Briefly define the axiom of *independence from irrelevant alternatives* and explain why it is considered necessary in Arrow's theory.

Briefly define the *weak order axiom* and explain why it is considered necessary in Arrow's theory.