(1) Verify if the following linear systems are consistent or not

$$(a)\begin{cases} x+y-3z=6 \\ 3x-y+2z=3 \\ -x+2y-z=1 \end{cases} \quad (b)\begin{cases} 2x-y+2z=2 \\ x+3y-z=8 \\ -x+4y-3z=6 \end{cases}$$

(2) Discuss the nature of the following linear system by varying the real parameter α,

$$\begin{cases} x-y-3w=-1 \\ 4x+3y+2z+w=1 \\ 5x+\alpha y+z+2w=0 \\ 2x+z+2w=0 \end{cases}$$

(3) Discuss the nature of the following linear system by varying the real parameters α and β

$$\begin{cases} x+2y+\alpha z=1 \\ 2x+\alpha y+8z=-1 \\ 4x+7y+z=\beta \end{cases}$$

(4) Determine whether the following vectors are linearly independent in $\mathbb{R}^3$,

$$\left\{\begin{pmatrix}0\\1\\-1\end{pmatrix},\begin{pmatrix}2\\0\\1\end{pmatrix}\right\};\ \left\{\begin{pmatrix}1\\2\\3\end{pmatrix},\begin{pmatrix}0\\4\\5\end{pmatrix},\begin{pmatrix}6\\7\\8\end{pmatrix}\right\};$$

$$\left\{\begin{pmatrix}1\\1\\3\end{pmatrix},\begin{pmatrix}-1\\2\\1\end{pmatrix},\begin{pmatrix}0\\6\\8\end{pmatrix}\right\},\ \left\{\begin{pmatrix}1\\t\\t^2\end{pmatrix},\begin{pmatrix}1\\t-1\\(t-1)^2\end{pmatrix},\begin{pmatrix}1\\(t+1)\\(t+1)^2\end{pmatrix}\right\}.$$

Are a basis for $\mathbb{R}^3$ (linear combinations fill $\mathbb{R}^3$)?

(5) Let

$$\mathbf{v_1} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}, \ \mathbf{v_2} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix},$$

and $B = \{\mathbf{v_1}, \mathbf{v_2}\}$, $V = Span\{\mathbf{v_1}, \mathbf{v_2}\}$. Is B a basis for V? Determine if $\mathbf{v}$ is in V , and if it is, find the coordinate vector of $\mathbf{v}$ relative to B.

(6) Determine the rank of the matrix

$$A = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$$

(7) Suppose we divide the economy (of a region) into n sectors, we indicate with $\mathbf{x} \in \mathbb{R}^n$ the *production vector*: output of each sector for year. Also let $\mathbf{d} \in \mathbb{R}^n$ be the *demand vector*: value of goods and services demanded from sectors by non-productive part of economy. If the *intermediate demand* are the inputs producers need for production, the Leontief's question: is there a production level such that the total amount produced equals the total demand for production? We assume that

- hold prices of goods and services constant;
- measure unit of input and output in millions of euro;
- for each sector, there is a unit consumption vector $\mathbf{c}$ listing inputs needed per unit of output of sector.

Suppose that $n = 3$ and that the table of intermediate values is as in table 1. The final demand is $d_1 = 50$ units for manufacturing, $d_2 = 30$ units for agriculture, and $d_3 = 20$ units for services. Build and discuss the corresponding input/output model.

Purchased from	Manufactoring	Agriculture	Services
Manufactoring	0.50	0.40	0.20
Agriculture	0.20	0.30	0.10
Services	0.10	0.10	0.30

tab6i.2

TABLE 1. Inputs Consumed per Unit of Output.