4

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix} \qquad \lambda = 2 \, ?$$

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix} - \lambda I \Rightarrow \begin{bmatrix} 1 & 4 & 2 \\ 1 & 4 & 2 \\ 1 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 2 \\ 1 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 \leftarrow R_1 - R_3$

$$\sim \begin{bmatrix} 1 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 4x_2 + 2x_3 = 0$

$x_1 = -4x_2 - 2x_3$

x_2 FREE x_3 FREE

$x_2 = -2x_3 - x_1$

$$V = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} x_3$$

THE VECTOR $[-4 \ 1 \ 0]^T$, $[-2 \ 0 \ 1]^T$

ARE LINEARLY INDEP. AND PROVIDE BASIS

DIM OF EIGENSPACE IS 2!

FREE VARIABLE?

5

$$\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} \quad \lambda ?$$

$$Av = \lambda v \qquad \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} v = \lambda v$$

$$\begin{bmatrix} 3-\lambda & 2 \\ 2 & 6-\lambda \end{bmatrix} = (3-\lambda)(6-\lambda) - 4 =$$

$$18 + \lambda^2 - 3\lambda - 6\lambda - 4 =$$

$$= \lambda^2 - 9\lambda + 14$$

$$\lambda_1, \lambda_2 = \frac{9 \pm \sqrt{81-56}}{2} = \frac{9 \pm \sqrt{25}}{2}$$

$$= \frac{9 \pm 5}{2} \begin{matrix} \nearrow 7 \\ \searrow 2 \end{matrix}$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} - 7\lambda = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} -4 & 2 \\ 0 & 0 \end{bmatrix}$$

$$-4x_1 + 2x_2 = 0$$

$$4x_1 = 2x_2$$

$$x_1 = \frac{1}{2} x_2$$

$$v_1 = \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} x_2$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} - 2\lambda = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \sim$$

$$\sim \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

$$v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} x_2$$

$$M = U \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} U^T$$

$$U = \begin{bmatrix} \frac{1}{2} & -2 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \frac{1}{2} & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \\ -2 & 1 \end{bmatrix}$$

no!

$$M = \begin{bmatrix} \frac{1}{2} \cdot 7 + 0 & 0 - 4 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{2} & -4 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 \\ -2 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{7}{2} \cdot \frac{1}{2} + -4 \cdot -2 & \frac{7}{2} - 4 \\ 7 \cdot \frac{1}{2} - 4 & 7 + 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{7}{4}+8 & \frac{2}{2}-4 \\ -\frac{1}{2} & 9 \end{bmatrix} = \begin{bmatrix} \frac{39}{4} & -\frac{1}{2} \\ -\frac{1}{2} & 9 \end{bmatrix}$$

No!

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot 7 + 0 & 0 - 4 \\ 14 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ 14 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -4 \\ 14 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 7+8 & 14-4 \\ & \end{bmatrix}$$

$$\begin{bmatrix} 14-4 & 28+2 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 \\ 10 & 39 \end{bmatrix} - 5\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$$

we get a scalar of M

U orthogonal matrix

$$\left\| \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\| = \sqrt{1+2^2} = \sqrt{5}$$

$$a = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

$$a \cdot a^T = a^T a = I$$

$$a \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix} a^T = M$$

$$D = \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$$

1)

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$(2-\lambda)^2 - 1 =$$

$$\lambda^2 - 4\lambda + 4 - 1 = \lambda^2 - 4\lambda + 3$$

$$\lambda_1, \lambda_2 = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$= \frac{4 \pm 2}{2} = 2 \pm 1$$

$$\lambda \begin{matrix} \nearrow 3 \\ \searrow 1 \end{matrix}$$

$$v_1 = \begin{bmatrix} 2-3 & -1 \\ -1 & 2-3 \end{bmatrix} =$$

$$= \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$x_1 = -x_2$$

$$v_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} x_2$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_1 - x_2 = 0 \quad x_1 = x_2$

$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_2$

V_1 AND V_2 ORTHOGONAL BECAUSE A IS SYMMETRICAL AND $\lambda_1 \neq \lambda_2$

$AV = \lambda v \Rightarrow A^2 v = \lambda^2 v$

$A^2 \rightarrow \lambda^2 \quad \lambda_1^2 = 9 \quad \lambda_2^2 = 1$

$A^{-1} A v = \lambda v A^{-1} \Rightarrow \frac{1}{\lambda} v = A^{-1} v$

$A^{-1} \rightarrow 1/\lambda \quad \frac{1}{\lambda_1} = 1/3 \quad \frac{1}{\lambda_2} = 1$

$A + 4I$

$(A+4I)v = Av + 4Iv$

$\Rightarrow (A+4I)v = v(A+4I)$

$\Rightarrow \lambda v + 4v \Rightarrow$

$(A+4I)v = (\lambda+4)v$

$A + 4I$ $\qquad \lambda_1 + 4 = 7$

$\qquad \lambda_2 + 4 = 5$

$\det \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 4 - 1 = 3$

$\lambda_1 \cdot \lambda_2 = 3$ ✓

② $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$

$$\begin{bmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{bmatrix} = (3-\lambda)(2-\lambda) - 2$$

$$= 6 - 5\lambda + \lambda^2 - 2 = \lambda^2 - 5\lambda + 4$$

$$\lambda_1, \lambda_2 = \frac{5 \pm \sqrt{25-16}}{2} = \begin{cases} 4 \\ 1 \end{cases}$$

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \quad x_1 = x_2$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x_2$$

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \qquad 2x_1 + x_2 = 0$$

$$x_1 = -\frac{x_2}{2}$$

$$v_2 = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix} x_2$$

$$\begin{bmatrix} 1 & -1/2 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & \frac{3}{2} \end{bmatrix}$$

2 pivot

2 ind. vectors

$R_2 \leftarrow -R_1 + R_2$

$$\det = -\frac{1}{2} - 1 \overset{!}{=} 0$$

3

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \quad \lambda = 5$$

$$\begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \sim \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} \qquad -2x_1 + x_2$$

$$x_1 = \frac{1}{2} x_2$$

$$v_1 = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} x_2$$

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \quad \lambda = -1$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \sim \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

$$2x_1 + 3x_2 = 0$$

$$x_1 = -\frac{3}{2} x_2$$

$$v = \begin{bmatrix} -\frac{3}{2} \\ 1 \end{bmatrix} x_2$$

(4) $$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 2 \\ 1 & 4 & 4 \end{bmatrix} \quad \lambda = 2$$

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 4 & 2 \\ 1 & 4 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

$$x_1 = -2x_2 - x_3$$

$$v = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} x_3$$

$$\begin{bmatrix} 1 & 3 & \alpha & 1 \\ 0 & 1 & \alpha & -1 \\ 0 & 0 & -3\alpha(\alpha-6) & 2(\alpha-4) \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\left| \begin{matrix} 1 & 3 & \alpha & 1 & 0 \\ 0 & 1 & \alpha & -1 & 1 \\ 0 & 0 & -3\alpha(\alpha-6) & 2(\alpha-4) & 0 \end{matrix} \right|$$

$1 \quad 3$

$$\left| \begin{matrix} 3 & 2 \\ 2 & 6 \end{matrix} \right| \quad (3-\lambda)(6-\lambda) - 4 =$$

$$= 18 + \lambda^2 - 9\lambda - 4 = \lambda^2 - 9\lambda + 14$$

$$\lambda_{1,2} = \frac{9 \pm \sqrt{81 - 56}}{2} = \frac{9 \pm 5}{2} \begin{cases} 7 \\ 2 \end{cases}$$

$$\begin{vmatrix} -4 & 2 \\ 2 & -1 \end{vmatrix} \sim \begin{vmatrix} -2 & 1 \\ 0 & 0 \end{vmatrix} \quad -2x_1 + x_2 \Rightarrow$$

$$x_1 = \frac{x_2}{2}$$

$$v_1 = \begin{vmatrix} 1/2 \\ 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} \quad x_1 + 2x_2 \Rightarrow$$

$$x_1 = -2x_2$$

$$v_2 = \begin{vmatrix} -2 \\ 1 \end{vmatrix}$$

$$U = \begin{vmatrix} 1/2 & -2 \\ 1 & 1 \end{vmatrix} \qquad U^T = \begin{vmatrix} 1/2 & 1 \\ -2 & 1 \end{vmatrix}$$

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$$M = \begin{vmatrix} 1/2 & -2 \\ 1 & 1 \end{vmatrix} \begin{vmatrix} 7 & 0 \\ 0 & 2 \end{vmatrix} \begin{vmatrix} 1/2 & 1 \\ -2 & 1 \end{vmatrix}$$

$$w_1 = \sqrt{\left(\tfrac{1}{2}\right)^2 + 1^2} = \sqrt{\tfrac{1}{4} + 1}$$

$$\sqrt{\frac{1+4}{4}} = \sqrt{\frac{5}{4}} =$$

$$\frac{\sqrt{5}}{2}$$

$$\frac{1}{4}\left(1 + 4\right)$$

$$\frac{1}{2}\sqrt{1+4}$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{5}} \end{bmatrix} \cdot \begin{bmatrix} 7 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{vmatrix} \frac{7}{\sqrt{3}} & -\frac{1}{\sqrt{5}} \\ \frac{14}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{vmatrix}$$

$$\begin{vmatrix} 7 & -4 \\ 14 & 2 \end{vmatrix} * \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 7+8 & 14-4 \\ 10 & 30 \end{vmatrix} = \begin{vmatrix} 15 & 10 \\ 10 & 30 \end{vmatrix}$$

$$5\begin{vmatrix} 3 & 2 \\ 2 & 6 \end{vmatrix}$$

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$$\begin{vmatrix} 2 & -2 & 0 & 0 \\ 1 & 2 & 2 & 1 \\ \alpha & -\alpha & 1 & 1 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 2 & -2 & 0 & 0 \\ \alpha & -\alpha & 1 & 1 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & -6 & -4 & -2 \\ 0 & 3\alpha & 2\alpha-1 & \alpha-1 \end{vmatrix}$$

$R_2 \leftarrow -R_1 \cdot 2 + R_2$

$R_3 \leftarrow R_1\alpha - R_3$

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 6 & 4 & 2 \\ 0 & \alpha & 2\alpha-1 & \alpha-1 \end{vmatrix} \sim \begin{vmatrix} 0 & 6 & & \\ 0 & 0 & -\frac{4}{3}\alpha & -\frac{4}{6}\alpha \end{vmatrix}$$

$$R_3 \leftarrow \alpha \frac{1}{6} R_2 - R_3$$

$\frac{2}{6}\alpha - \alpha + 1$

$\frac{1}{6} \cdot 4\alpha - (2\alpha - 1)$

$\frac{2-6}{6}$

$\frac{1}{3} 2\alpha - 2\alpha + 1$

$-\frac{4}{6}\alpha - 1$

$\frac{2-6}{3}\alpha + 1$

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 6 & 4 & 2 \\ 0 & 0 & -\frac{4}{3}\alpha & -\frac{2}{3}\alpha \end{vmatrix}$$

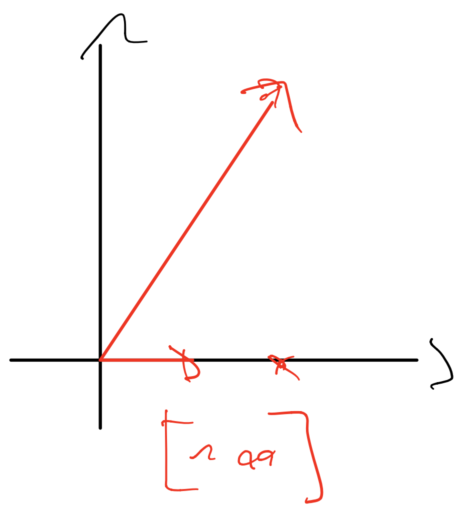
$\vec{c} = 0 \quad [0\,0\,0\,0]$

d)

② $A = \left\{ \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \right\}$

$$\begin{vmatrix} 0 & 3 \\ 2 & 0 \\ 1 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 3 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 \\ 0 & 4 \\ 0 & 5 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 \\ 0 & 4 \\ 0 & 0 \end{vmatrix}$$



DIP

$$x_1 + 2x_2 = 0 \qquad x_1 = -2x_2$$

$$4x_2 = 0 \qquad x_2 = 0$$

$$x_2 = x_1 = 0 \quad \text{IND.}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 0 \\ 1 & 2 & 0 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$R_2 \leftarrow R_2 - R_1$$

$$\sim \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \text{IND.}$$

3)
$$\begin{vmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

2 → FREE VARIABLE

COL-RANK $4-2=2$

5) $x \begin{vmatrix} 1 \\ 1 \end{vmatrix}$ $y \begin{vmatrix} -2 \\ 4 \end{vmatrix}$

$$\text{PROJ} = \frac{|x \cdot y|}{|x|^2} y$$

$$P = \frac{-2}{\;} \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$x = 2$

$v = 1$ $\sqrt{2 - 1} = 1$

$\|v\| = 1$

6

$$\begin{vmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 1 & 3 \end{vmatrix} \qquad \lambda = 3$$

$$\begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 4 & 1 & 0 \end{vmatrix} \sim \begin{vmatrix} 4 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{vmatrix} \sim$$

$$\begin{vmatrix} 4 & 1 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 \end{vmatrix} \sim \begin{vmatrix} 4 & 1 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 4 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$4x_1 \times x_2 = 0$

$x_2 = 0 \qquad x_1 = \frac{1}{4} x_2$

x_3 frei

$$v = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} x_3 + \begin{vmatrix} 1/4 \\ 1 \\ 0 \end{vmatrix} x_2$$

$$\begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} = (3-\lambda)^2 - 4 =$$

$$\lambda^2 + 9 - 6\lambda - 4 =$$

$$\lambda^2 + 5 - 6\lambda \Rightarrow$$

$$\frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2}$$

$$3 \pm 2 \left\langle \begin{matrix} 1 \\ 5 \end{matrix} \right.$$

$$v_1) \begin{vmatrix} 2 & -2 \\ -2 & 2 \end{vmatrix} \sim \begin{vmatrix} 2 & -2 \\ 0 & 0 \end{vmatrix}$$

$$x_1 = x_2$$

$$v_1 \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$

$$v_2) \begin{vmatrix} -2 & -2 \\ -2 & -2 \end{vmatrix} \simeq \begin{vmatrix} -1 & -1 \\ 0 & 0 \end{vmatrix}$$

$$-x_1 - x_2 = 0 \qquad x_1 = -x_2$$

$$v_2 = \begin{vmatrix} -1 \\ 1 \end{vmatrix} x_2$$

Prova N. 2

$$\begin{vmatrix} 1 & -3 & 2 & -4 \\ -3 & -9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{vmatrix} \sim \begin{vmatrix} 1 & -3 & 2 & -4 \\ 0 & -18 & 5 & -7 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & 6 & -9 \end{vmatrix}$$

$R_2 \leftarrow R_1 \cdot 3 + R_2$

$R_3 \leftarrow R_1 \cdot 2 - R_3$

$R_4 \leftarrow R_1 \cdot 4 + R_4$

$$\sim \begin{vmatrix} 1 & -3 & 2 & -4 \\ 0 & -18 & 5 & -7 \\ 0 & 0 & 6 & -9 \\ 0 & 0 & 0 & -5 \end{vmatrix}$$

d)

2)

$$\begin{vmatrix} -t & (t-1) & 1 & 1 \\ 0 & t-1 & t & 1 \\ 2 & 0 & 1 & 5 \end{vmatrix} \sim \begin{vmatrix} 2 & 0 & 1 & 5 \\ -t & (t-1) & 1 & 1 \\ 0 & t-1 & t & 1 \end{vmatrix}$$

$$\sim \begin{vmatrix} 2 & 0 & 1 & 5 \\ 0 & t-1 & \frac{t}{2}+1 & \frac{5t}{2}+1 \\ 0 & t-1 & t & 1 \end{vmatrix}$$

$R_2 \leftarrow R_1 \frac{t}{2} + R_2$

$\frac{2b}{2} + -t$ $\qquad$ $\frac{t}{2} + t$

$$\frac{t}{2}+1$$

$$\begin{vmatrix} 2 & 0 & 1 & 5 \\ 0 & t-1 & \frac{t}{2}+1 & \frac{5t}{2}+1 \\ 0 & t-1 & t & 1 \end{vmatrix} \sim \begin{vmatrix} 2 & 0 & 1 & 5 \\ 0 & t-1 & \frac{t}{2}+1 & \frac{5t+1}{2} \\ 0 & 0 & -\frac{t}{2}+1 & \frac{5t}{2} \end{vmatrix}$$

$$\begin{vmatrix} 2 & 0 & 1 & 5 \\ 0 & (t-1) & \left(\frac{t}{2}+1\right) & \frac{5t+1}{2} \\ 0 & 0 & \left(-\frac{t}{2}+1\right) & \frac{5t}{2} \end{vmatrix}$$

$t=2$

$$\begin{bmatrix} 0 & 0 & 0 & \cancel{0} \end{bmatrix}$$

$t=1$

$$\begin{vmatrix} 2 & 0 & 1 & 5 \\ 0 & 0 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{vmatrix} \sim \begin{bmatrix} 2 & 0 & 1 & 5 \\ 0 & 0 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 & -\frac{8}{6} \end{bmatrix}$$

$R_3 \leftarrow R_2 - R_3$

$$\frac{7}{6} - \frac{5}{2} \Rightarrow \frac{7 - \overset{3}{15}}{6} = -\frac{8}{6}$$

3) $$\begin{vmatrix} 2 & 3 & -1 \\ -8 & -7 & 6 \\ 6 & -1 & -7 \end{vmatrix} \sim \begin{vmatrix} 2 & 3 & -1 \\ 8 & 7 & -6 \\ -6 & 1 & 7 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 5 & 2 \\ 0 & 10 & 4 \end{vmatrix} \sim \begin{vmatrix} 2 & 3 & -1 \\ 0 & 5 & 2 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\left\{ \begin{vmatrix} 2 \\ -8 \\ 6 \end{vmatrix} \quad \begin{vmatrix} 3 \\ -7 \\ -1 \end{vmatrix} \right\}$$

RANK = 2

4

$$y_1 \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} \qquad y_2 \begin{vmatrix} 1 \\ 0 \\ -1 \end{vmatrix}$$

$$y_3 \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

$$c_1 \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} + c_2 \begin{vmatrix} 1 \\ 0 \\ -1 \end{vmatrix} + c_3 \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

$$\begin{matrix} 2 & -1 & 0 \\ 0 & 0 & 2 \\ 2 & 1 & 0 \end{matrix} = \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$$

5)

$$\begin{vmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{vmatrix} \qquad \lambda = 2$$

$$\begin{vmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{vmatrix} \sim \begin{vmatrix} 2 & -1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$2x_1 - x_2 + 6x_3 = 0 \qquad \begin{matrix} -5 \\ 2 \\ 2 \end{matrix}$$

$$(2 \cdot -5) - 2 + 6 \cdot 2$$

$$-10 - 2 + 12 = 0$$

$$\begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix} = (7 - \lambda) \cdot (1 - \lambda) + 8$$

$$7 - 8\lambda + \lambda^2 + 8$$

$$\lambda^2 - 8\lambda + 15$$

$$\frac{8 \pm \sqrt{64-60}}{2} = 4 \pm 1 \begin{cases} 5 \\ 3 \end{cases}$$

$$\begin{vmatrix} 2 & 2 \\ -4 & -4 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$$

$$x_1 = -x_2$$

$$v_1 = \begin{vmatrix} -1 \\ 1 \end{vmatrix} x_2$$

$$\begin{vmatrix} 4 & 2 \\ -4 & -2 \end{vmatrix} \sim \begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix}$$

$$2x_1 = -x_2 \qquad x_1 = -\frac{1}{2} x_2$$

$$v_2 = \left|\begin{matrix} -1/2 \\ 1 \end{matrix}\right| x_2$$

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$$\left|\begin{matrix} 4 & -2 & 3 \\ 0 & -1 & 3 \\ -1 & 2 & -2 \end{matrix}\right| \quad \lambda = 1$$

$$\left|\begin{matrix} 3 & -2 & 3 \\ 0 & -2 & 3 \\ -1 & 2 & -3 \end{matrix}\right| \sim \left|\begin{matrix} 1 & -2 & 3 \\ 3 & -2 & 3 \\ 0 & -2 & 3 \end{matrix}\right|$$

$$\sim \left|\begin{matrix} 1 & -2 & 3 \\ 0 & -4 & 6 \end{matrix}\right| \sim \left|\begin{matrix} 1 & -2 & 3 \\ 0 & -4 & 6 \end{matrix}\right|$$

$$\begin{vmatrix} 0 & -2 & 3 \end{vmatrix} \quad \begin{vmatrix} 0 & 0 & 0 \end{vmatrix}$$

$$x_1 - 2x_2 + 3x_3$$

$$-4x_2 + 6x_3 \qquad x_2 = \frac{3}{2}x_3$$

$$x_1 = 2x_2 - 3x_3 \Rightarrow 0$$

$$v = \begin{bmatrix} 0 \\ 3/2 \\ 1 \end{bmatrix} x_3$$

$$\begin{vmatrix} 4 & -2 & 3 \\ 0 & -1 & 3 \\ -1 & 2 & -2 \end{vmatrix} \quad \begin{vmatrix} 6 \\ \frac{3}{2} \\ 1 \end{vmatrix}$$

$$0 \quad -3 \quad +3$$

$$0 \quad -\tfrac{3}{2} \quad +3$$

$$3 \quad \cancel{-}2$$

$$\begin{vmatrix} 0 \\ \frac{3}{2} \\ 1 \end{vmatrix} 1$$

$$Av = \lambda v$$

---

3)

$$\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} \qquad \lambda_{1,2} = 1/3$$

$$\begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix} \sim \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 \\ 2 & -2 \end{vmatrix} \sim \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}$$

$$v_1 = \begin{vmatrix} 0 \\ 1 \end{vmatrix} x_2$$

$$v_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix} x_2$$

$$(3-\lambda)(1-\lambda)$$

$$3 - 4\lambda + \lambda^2$$

$$\frac{4 \pm \sqrt{16-12}}{2} = 2 \pm 1 \begin{cases} 3 \\ 1 \end{cases}$$

$$\begin{vmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{vmatrix}$$

$$\sim \begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 5 & 1 & 2 & 2 & 0 \end{vmatrix} \sim \begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 1 & 9 \\ 0 & 0 & 0 & 5 & -16 \\ 0 & \frac{3}{2} & \frac{1}{2} & -2 & -5 \end{vmatrix}$$

$R_2 \leftarrow R_1 \frac{3}{2} - R_2$

$R_3 \leftarrow R_1 4 - R_3$

$R_4 \leftarrow R_1 \frac{5}{2} - R_4$

$\frac{5}{2} - \frac{2}{2}$

$$\sim \begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 1 & 9 \\ 0 & \frac{3}{2} & \frac{1}{2} & -2 & -5 \\ 0 & 0 & 0 & 5 & -20 \end{vmatrix}$$

$$\sim \begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 1 & 9 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 5 & -2 \end{vmatrix}$$

$R_3 \leftarrow R_2 + R_3$

$$\begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 1 & 9 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 5 & -20 \end{vmatrix} \sim \begin{vmatrix} 2 & 1 & 1 & 0 & -2 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & 1 & 9 \\ 0 & 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$R = 3$

$$\left|\begin{matrix} 1 & 1 & -3 & 6 \\ 3 & -1 & 2 & 3 \\ -1 & 2 & -1 & 1 \end{matrix}\right| \sim$$

$$\left|\begin{matrix} 1 & 1 & -3 & 6 \\ 0 & 4 & -11 & 15 \\ 0 & 3 & -4 & 7 \end{matrix}\right| \sim$$

$R_3 \leftarrow R_3 + R_1$

$R_2 \leftarrow R_1 \cdot 3 - R_2$

$$\left|\begin{matrix} 1 & 1 & -3 & 6 \\ 0 & 4 & -11 & 15 \\ 0 & 0 & \frac{39}{4} & \end{matrix}\right|$$

$$\frac{3}{4}R_2 - R_3$$

$$\frac{33}{4} + \frac{16}{4}$$

$$\left|\begin{matrix} 2 & -1 & 2 & 2 \\ 1 & 3 & -1 & 8 \\ -1 & 4 & -3 & 6 \end{matrix}\right| \sim \left|\begin{matrix} 1 & 3 & -1 & 8 \\ 2 & -1 & 2 & 2 \\ -1 & 4 & -3 & 6 \end{matrix}\right|$$

$$\left|\begin{matrix} 1 & 3 & -1 & 8 \\ 0 & -7 & -3 & 14 \\ 0 & 7 & -4 & 14 \end{matrix}\right| \sim$$

The Pythagorean Theorem

Two vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if and only if $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$.

$$\|u\|^2 = 0 + \alpha^2 + \beta^2$$

$$\|v_2\|^2 = \beta^2 - \alpha^2$$

$$\|v_3\|^2 = 1$$

$$v_1 + v_2 \quad \begin{bmatrix} \beta \\ -2\alpha \\ \beta \end{bmatrix}$$

$$\beta^2 + \beta^2 = 2\beta^2$$

$$v_1 v_3 \quad \begin{vmatrix} 0 \\ \alpha + 1 \end{vmatrix}$$

$$\begin{vmatrix} a+1 \\ B \end{vmatrix}$$

$$(a+1)$$

$$\begin{vmatrix} 2 & -2 & 0 & 0 \\ 1 & 2 & 2 & 1 \\ a & -a & 1 & 1 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 2 & -2 & 0 & 0 \\ a & -a & 1 & 1 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 6 & 4 & 2 \\ 0 & 3a & 2a-1 & a-1 \end{vmatrix}$$

$$R_2 \leftarrow R_1 2 - R_2$$

$$R_3 \leftarrow aR_1 - R_3$$

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 2a & 2a-1 & a-1 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & -1 & -1 \end{vmatrix}$$

$$R_3 \leftarrow R_2\, a - R_3$$

$$2a - 2a - 1$$

$$a - a - 1$$

$$\begin{vmatrix} 0 & 3 \\ 2 & 0 \\ 1 & 2 \end{vmatrix}$$

L_i l.

$x_1 = 0 \qquad x_1 = x_2$

$x_2 = 0$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 0 \\ 1 & 2 & 0 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad 4.\,1.$$

3) $$\begin{vmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{vmatrix} \sim$$

$$\begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -2 & -4 \end{vmatrix} \sim \begin{vmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

$$R_2 \leftarrow R_1 \, 2 - R_2$$

2 FREE VAR $\rightarrow$ DIM$(N(A))=2$

4) $\begin{vmatrix} x_1 + x_2 - x_3 \\ x_1 + x_3 \end{vmatrix}$

$$\begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}$$

5) $x \begin{vmatrix} 1 \\ 1 \end{vmatrix}$ $\quad y = \begin{vmatrix} -2 \\ 0 \end{vmatrix}$

$$\text{Proj}_y x = \frac{x \cdot y}{\|y\|} y$$

$$\|y\| = -2^2 + 0 = 4$$

$x \cdot y = 1 \cdot -2 + 0 = -2$

$\text{proj}_x y = \frac{-2}{4} \begin{pmatrix} -2 \\ 0 \end{pmatrix} =$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|x| = 1 + 1 = 2$

$\text{proj}_y x = \frac{-2}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$C = R$

$\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$

$$Av = \lambda v$$

$$\begin{vmatrix} 3-\lambda & -2 \\ -2 & 3-\lambda \end{vmatrix}$$

$$(3-\lambda)^2 - 4 = \lambda^2 + 9 - 6\lambda - 4$$

$$\lambda^2 - 6\lambda + 5$$

$$\lambda_1, \lambda_2 = \frac{6 \pm \sqrt{36-20}}{2} =$$

$$= \frac{6 \pm 4}{2} = 3 \pm 2 \begin{cases} 5 \\ 1 \end{cases}$$

$$(A - I)v_2 = 0$$

$$\begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} \sim \begin{vmatrix} 1 & -1 \\ 0 & 0 \end{vmatrix}$$

$$x_1 = x_2$$

$$v_1 = \begin{vmatrix} 1 \\ 1 \end{vmatrix} x_2$$

$$(A - 5I)v_1 = 0$$

$$\begin{vmatrix} -2 & -2 \\ -2 & -2 \end{vmatrix} \sim \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$$

$x_1 = -x_2$

$$v_2 = \begin{vmatrix} -1 \\ 1 \end{vmatrix} x_2$$

$$\frac{3}{\sqrt{11}} \cdot -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{11}} \frac{3}{\sqrt{6}}$$

$+ \ 1$

$$\begin{vmatrix} 1 & 3 & a & 1 \\ 2 & a & 0 & 6 \\ 1 & 4 & 2a & 0 \end{vmatrix} \ r$$

$$\begin{vmatrix} 1 & 3 & a & 1 \\ 0 & 6-a & 2a & 2-a \\ 0 & -1 & -a & 1 \end{vmatrix} \sim$$

$$\sim \begin{vmatrix} 1 & 3 & a & 1 \\ 0 & -1 & -a & 1 \\ 0 & 6-a & 2a & 2-a \end{vmatrix}$$

$$\left| \begin{matrix} \\ \\ 0 & -1 \\ 0 & 0 \end{matrix} \right.$$

$$-a(6-a)-2a$$

$$-a\,8a+a^2$$

$$a(a-8)$$

$$a(6-a)-2a$$

$$6a-a^2-2a$$

$$4a-a^2$$

$$a(4-a)$$

$x = A^{-1} b$

$Ax = b$

$A = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$ $\quad (2-\lambda)^2 - 1 =$

$\lambda^2 - 4\lambda + 4 - 1 =$

$\lambda^2 - 4\lambda + 3 \qquad \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2}$

$= 2 \pm 1 \begin{cases} 3 \\ 1 \end{cases}$

$-1 \quad -1 \qquad x_1$

$\lambda = 3 \qquad \lambda_1$

$$\lambda_1 \cdot \lambda_2 = \det(A)$$

$$\lambda_1^2 \cdot \lambda_2^2 = \det(A^2)$$

$$x \begin{vmatrix} 1 \\ 1 \end{vmatrix} \qquad y \begin{vmatrix} -2 \\ 0 \end{vmatrix}$$

$$x \to y$$

$$\frac{x \cdot y}{\|y\|} \cdot y \qquad x \cdot y = 1 \cdot -2 + 0 \cdot 1 = -2$$

$$|y| = -2^2 + 0^2 = 4$$

$$\cdot \frac{1}{2} \begin{vmatrix} -2 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

$$\|x\| = 1^2 + 1^2 = 2$$

$$-\frac{2}{2} \begin{vmatrix} 1 \\ 1 \end{vmatrix} \to \begin{vmatrix} -1 \\ -1 \end{vmatrix}$$