PREFERENCE STRUCTURING

$|\Omega| = 1$

$|D| = 1$

$\precsim$ REFLEXIVE TRANSITIVE | PRE-ORDER | WEAK-ORDER

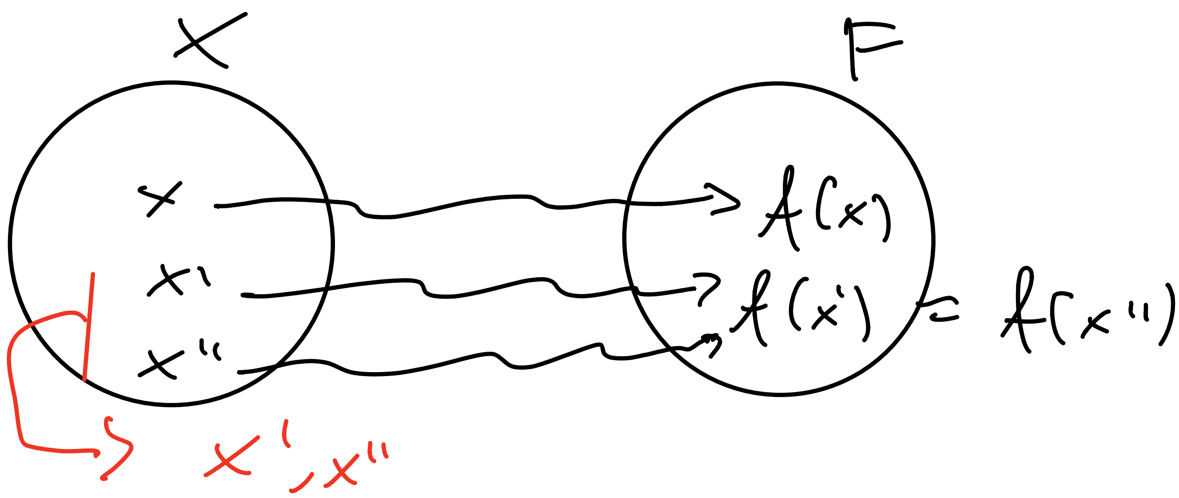
PREFERENCE

$f(x) \precsim f(x')$

$\Updownarrow$

$x \precsim x'$ DOMINANCE

COMPLETE



ANTISYMMETRY

$f \leq f'$ AND $f' \leq f \Rightarrow f = f'$

$x \leq f'$ AND $x' \leq x \Rightarrow x = x'$

STRICT DOMINANCE

$$x < x' \Leftrightarrow f(x) < f(x')$$

NON DOMINATED SOLUTIONS

$$X_{ND} = \{ x \in X : \nexists\, x' \in X \mid x' < x \}$$

① LEXICOGRAPHIC ORDER

$$f \underset{LEX}{\leq} f' \Leftrightarrow (f_{\pi_1} < f'_{\pi_1}) \text{ OR}$$

$$((f_{\pi_1} < f'_{\pi_1}) \text{ AND } (f_{\pi_2} < f'_{\pi_2})) \text{ OR}$$

$$\ldots \left((f_{\pi_L} = f'_{\pi_L}) \left\{ \begin{array}{l} \forall L \notin R \setminus \{\pi_P\} \\ \text{AND} \\ (f_{\pi_P} < f'_{\pi_P}) \end{array} \right. \right)$$

EXAMPLE NAVIGATOR

X	COST	TIME
x_1	10	24
x_2	10	36
x_3	10	28
x_4	15	4
x_5	20	4
x_6	30	1

COST	TIME
f(IMPACT)	×

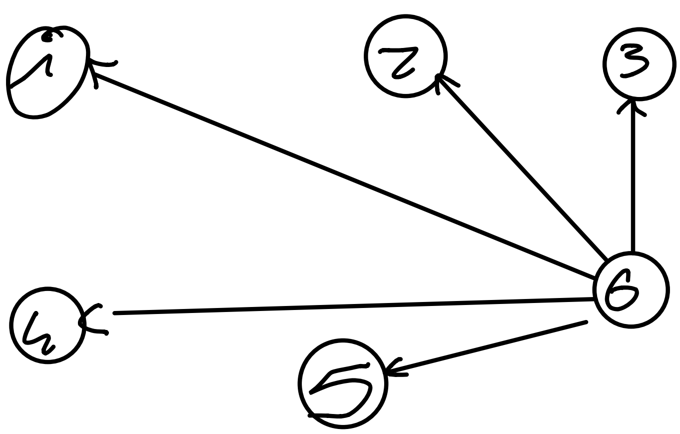
WHERE $(\pi_1, \pi_2 \dots \pi_p)$ IS A PERMUTATION OF P

$P = \{1, 2, \dots p\}$

LET (π_1, π_2) BE $(2, 1)$

WHEN x^4 PREFERRED x^5 $\quad 4 \longrightarrow 5$

WE HAVE $x^{(6)} < x^{(4)} < x^{(5)} < x^{(1)} < x^{(3)} < x^{(2)}$



IF COST PRIORITY:

WE HAVE ③ $x^{(1)}, x^{(2)}, x^{(3)}$ THE SAME

NO OPTIMAL SOLUTION

$$\min_{x \in X} f_{\tilde{\pi}_1}(x) \longrightarrow X^{c}_{\tilde{\pi}_1}$$

$$\min_{x \in X_{\tilde{\pi}_1}} f_{\tilde{\pi}_2}(x) \longrightarrow X_{\tilde{\pi}}$$

② LEXICOGRAPH ORDER WITH ASPIRATION LEVEL

THE SET

$$f \preceq_{ASP} f' \iff \left(f \in F_\varepsilon \text{ AND } f' \notin F_\varepsilon\right) \text{ OR}$$

$$\left(f \in F_\varepsilon \text{ AND } f' \in F_\varepsilon \text{ AND } f \leq_{LEX} f'\right)$$

WHERE $(\tilde{\pi}_1, \dots, \tilde{\pi}_p)$ BE A PERMUTATION OF

$P = \{1, 2 \dots p\}$

$$F_\varepsilon = \{ f \in F : f_{\tilde{\imath}_e} \le \varepsilon_e \ \forall e \in P \setminus \{\tilde{\imath}\} \}$$

EX FIND COST $\varepsilon = 18$

EXAMPLE NAVIGATOR

$x^4 < x^1 < x^3 < x^2 < x^5 < x^6$

x	COST	TIME
x_1	10	24
x_2	10	36
x_3	10	28
x_4	15	4
x_5	20	4
x_6	30	1

3 UTOPIA POINTS

OR another example in case we need to care about pollution or fixed cost

③ UPTOPIA POINT

$$f_\ell^0 = \min_{x \in X} f_\ell(x) \quad \ell \in P$$

we have a f_ℓ^0 that

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X	Cost	time
$x_{(1)}$	10	24
$x_{(2)}$	10	36
$x^{(3)}$	10	28
$x^{(4)}$	15	4
$x^{(5)}$	20	4
$x^{(6)}$	30	1
f_0	10	1 (Utopia point)

$f \prec f' \Leftrightarrow d(f; f^0) \leq d(f', f^0)$

where: $d(f, f')$ is a distance funct on F

$\min_{x \in X} d(f(x), f^0)$: tức tìm khoảng cách ngắn nhất của f để đạt vị trí utopia point kiểu đk ideal

$\rightarrow = \sqrt{(f_1(x) - 10)^2 + (f_2(x) - 1)^2} = L_2$

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X	Cost	time	$d(f_{cost}; f^0_{cost})$
$x_{(1)}$	10	24	0
$x_{(2)}$	10	36	0
$x^{(3)}$	10	28	0
$x^{(4)}$	15	4	0.25
$x^{(5)}$	20	4	1/5
$x^{(6)}$	30	1	1

khoảng cách cost đến cost ideal = 10

$\rightarrow$ consider $\max_{x_j} - \min_{f} = 1$

$\Rightarrow$ 30 − 10 = 20 → 1

10 ?

5 ?

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$d(f_{time}; f^0)$

Cost

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X	Cost	time	$d(f; f^\circ)_{time}$
	/		
$X_{(1)}$	10	24	23/35
$X_{(2)}$	10	36	1
$X^{(3)}$	10	28	27/35
$X^{(4)}$	15	4	3/35
$X^{(5)}$	20	4	3/35
$X^{(6)}$	30	1	0

(2)

$$\rightarrow \begin{cases} \max_{time} = 1 \\ \min_{time} = 0 \end{cases}$$

Compare $d(f; f^\circ)$ beetween time & cost, we see that cost you can get the number much smaller

$$\min_{x \in X} d(f(x), f^\circ) = \sqrt{[f_1(x) - 10]^p + (f_2(x) - 1)^p} = 4_p$$

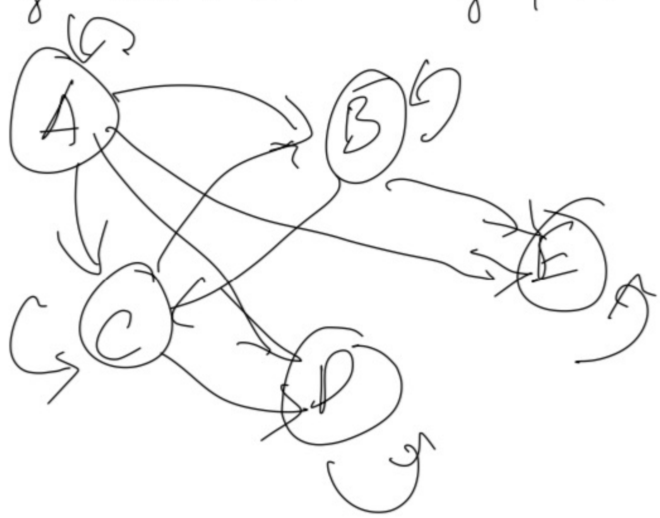
Mahattan distance in math

This uses for general indicator point

Suppose you has a problem:

X_{Finite}

If it's weak problem you can result with graph.



A is dominant solution

$X_{ND} = \{A\}$

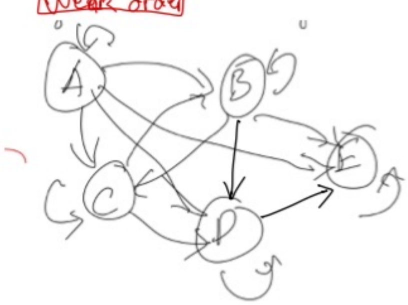
$v : F \rightarrow \mathbb{R}$ (value function)

$f \preceq f' \Leftrightarrow v(f) \geq v(f')$

BORDA's COUNT : French revolution

$B(f) = |\{f' \in F : f \prec f'\}|$

Weak order



f	$B(f)$
A	5
B	4
C	3
D	2
E	1

$\forall x_{ND} \rightarrow \max_{x \in X} B(f(x))$;

1 → weakness based on weak order graph.

④ MULTIPLE ATTRIBUTE UTILITY THEORY

In economics

We have { weak order π
$\exists\ v(\cdot)$ consistent with π $\quad v(\cdot)$: value function $= u(\cdot)$

(In this lesson write $u(\cdot)$ instead of $v(\cdot)$)

→ $\boxed{?}$ How to build $v(\cdot)$? (some time customer don't know $v(\cdot)$)
→ we have to build $v(\cdot)$ to reach the final purpose : $\boxed{\max_{x \in X} f(x)}$

$\boxed{\Rightarrow \text{ How to Build } u(\cdot)}$

Es. This happens also in video game development. How to choose a winner in a game? Which are characteristics to compare 2 Characters ? Indicators ?