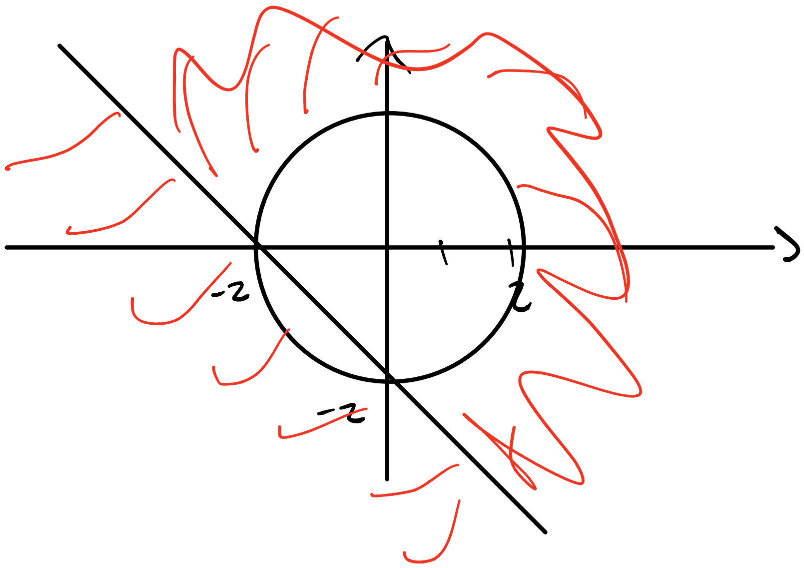
$$\min f(x) = x_1^2 + x_2^2$$

DIAMETRO

$$g_1(x) = x_1^2 + x_2^2 - 4 \leq 0$$

$$g_2(x) = -x_1 - x_2 - 2 \leq 0$$

$$x_2 = -x_1 - 2$$



NON REGULAR POINTS

$$\nabla g_1 = [\,2x_1 \quad 2x_2\,]$$

$$\nabla g_2 = [\,-1 \quad -1\,]$$

$$\nabla g_1 = 0 \longrightarrow \begin{cases} x_1 = 0 \\ x_2 = 0 \end{cases}$$

NON REGULAR

NON VALGONO CONDIZIONI KKT

$\nabla g_2 = 0$ MAI

PUNTI ATTIVI ? INTERSEZIONE

$$\begin{cases} x_1^2 + x_2^2 - 4 = 0 \\ x_1 + x_2 + 2 = 0 \end{cases} \qquad \begin{cases} (-x_2 - 2)^2 + x_2^2 - 4 = 0 \\ x_1 = -x_2 - 2 \end{cases}$$

$$\begin{cases} x_2^2 + \cancel{4} + 4x_2 + x_2^2 - \cancel{4} = 0 \end{cases}$$

$$\begin{cases} 2x_2^2 + 4x_2 = 0 \end{cases} \qquad 2x_2(x_2+2) = 0$$

$x_2 = 0$

$x_2 = -2$

$$\begin{cases} x_2 \begin{cases} 0 \\ -2 \end{cases} \\ x_1 = \begin{cases} -2 \\ 0 \end{cases} \end{cases}$$

$(-2, 0)$

$(0, -2)$

$A = (-2, 0) \qquad B(0, -2)$

VINCOLI ATTIVI IN QUESTI PUNTI

$A = (-2, 0)$

$\nabla g_1 = [\,2x_1 \quad 2x_2\,]$

$\nabla g_2 = [\,-1 \quad -1\,]$

$\nabla g_1\ [\,-4 \quad 0\,]$

$$M = \begin{bmatrix} -4 & -1 \\ 0 & -1 \end{bmatrix}$$

$\det(M) \neq 0$

L. IND $\Rightarrow$ PUNTI REGOLARI
VALE KKT

$B(0, -2)$

$\nabla g_{1B} = [\,-4 \quad 0\,]$

$$M_B \begin{bmatrix} 0 & -1 \\ -4 & -1 \end{bmatrix}$$

$\det(M_B) \neq 0$

Sono regular points!

CONDIZIONI KKT

$$L(x, \mu, \lambda)$$

$$f(x) + \cancel{\sum \lambda_i h_i} + \sum \mu_i g_i$$

vincoli con $g = 0$

$g \leq 0$

$$x_1^2 + x_2^2 + \mu_1(x_1^2 + x_2^2 - 4) + \mu_2(-\cancel{x_1} - x_2 - 2)$$

Punto ottimo con questo sistema

$$\begin{cases} 2x_1 + 2\mu_1 x_1 - \mu_2 \cdot 1 = 0 & \frac{dL}{dx_1} \\ 2x_2 + 2\mu_1 x_2 - \mu_2 \cdot 1 = 0 & \frac{dL}{dx_2} \\ \mu_1 g_1 = 0 \quad \mu_1(x_1^2 + x_2^2 - 2) = 0 \\ \mu_2 g_2 \quad \mu_2(-x_1 - x_2 - 2) = 0 \\ \mu_1 \geq 0, \mu_2 \geq 0 \quad g_1, g_2 \leq 0 \end{cases}$$

$$2x_1 - 2x_2 + 2\mu_1 x_1 - 2\mu_1 x_2$$

$$2(x_1 - x_2) + 2\mu_1(x_1 - x_2)$$

$2(x_1 - x_2)(1 + \mu_1) = 0$

$\mu_1 = -1$ non può esistere

$x_1 - x_2 = 0$

$$\begin{cases} 2x + 2\mu_1 x - \mu_2 \cdot 1 = 0 \\ 2x + 2\mu_1 x - \mu_2 \cdot 1 = 0 \\ \mu_1 g_1 = 0 \qquad \mu_1(x^2 + x^2 - 2) = 0 \\ \mu_2 g_2 \qquad \mu_2(-2x - 2) = 0 \\ \mu_1 \geq 0, \mu_2 \geq 0 \quad g_1, g_2 \leq 0 \end{cases}$$

1

$\mu_2 = 0$

$g_2 \leq 0$

2

$\mu_2 > 0$

$g_2 = 0$

1) $\Big(\; 2x + 2\mu_1 x = 0$

$$\begin{cases} 2x + 2\mu_1 x = 0 \\ \mu_1(2x^2 - 4) = 0 \\ \mu_2 = 0 \\ g_2 \leq 0 \end{cases}$$

$$\begin{cases} 2x(1+\mu_1) = 0 \\ \mu_1(x^2 - 2) = 0 \\ \mu_2 = 0 \\ g_2 \leq 0 \\ x^2 \leq 2 \quad g_2 \\ x \geq -1 \quad g_1 \end{cases}$$

$x = 0$

$O(0,0)$

(

2) $\begin{cases} g_2 = 0 \longrightarrow x_1 = -1 \\ x_2 = -1 \quad D(-1, -1) \end{cases}$

$f(0,0) \longrightarrow$ MIN!!

A. 2, 6 ESERCIZIO 3

$\max f_1 = x_1 + 3x_2$

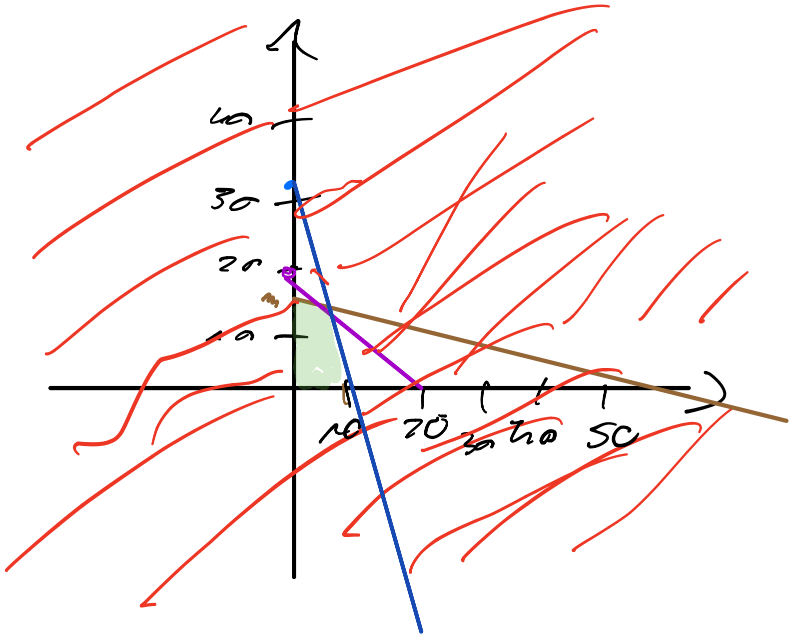
$\max f_2 = -3x_1 - 2x_2$

$2x_1 + x_2 \leq 32$

$x_1 + x_2 \leq 20$

$x_1 + 5x_2 \leq 72$

$x_1, x_2 \geq 0$



$y = \dfrac{72 - x_1}{5}$

0 → 14

10 → 12

20 → 10

$y = 20 - x_1$

0 — 20

10 — 10

20 — 0

$y = 32 - 2x_1$

0 → 32

10 → 12

20 → -10

$$\boxed{w(t_1, t_2) = 2t_1 + t_2}$$

$$w(t_1, t_2) = 2(x_1 + 3x_2) + -3x_1 - 2x_2 =$$

$$2x_1 + 6x_2 - 3x_1 - 2x_2 =$$

$$= -x_1 + 4x_2$$

$$O = \left(0, \frac{72}{5}\right)$$

$$2x_1 + x_2 = 32$$

$\max \ f_1 = -x_1 - x_2$

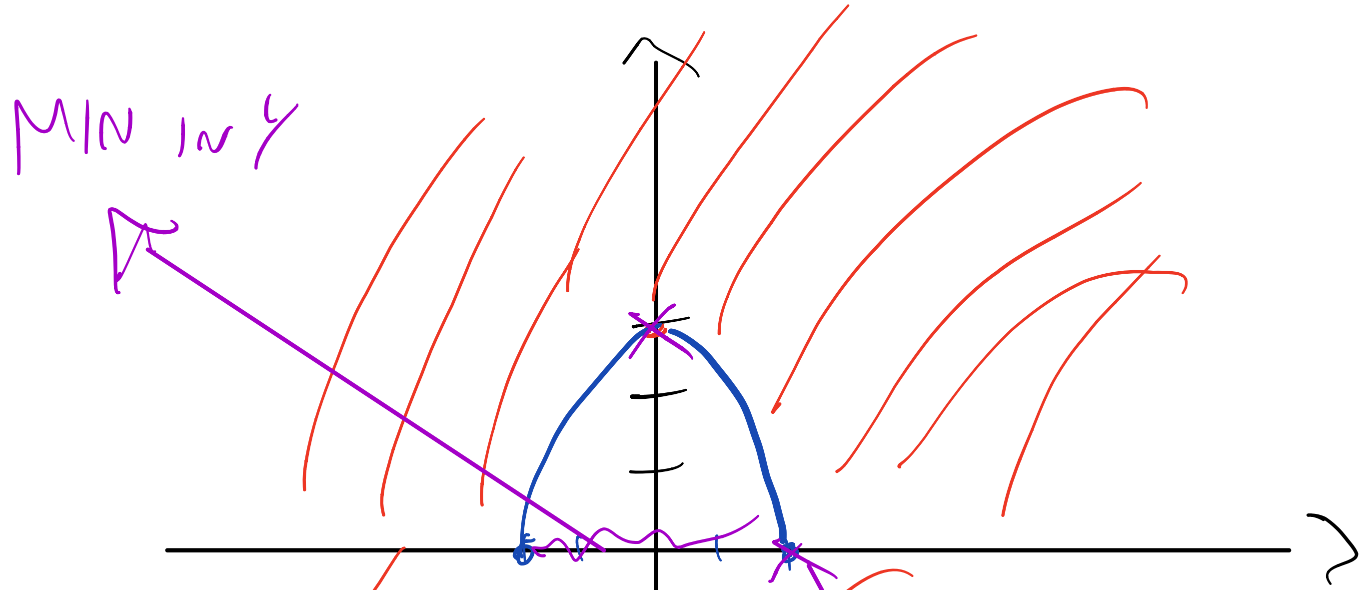
$\max \ f_2 = x_1$

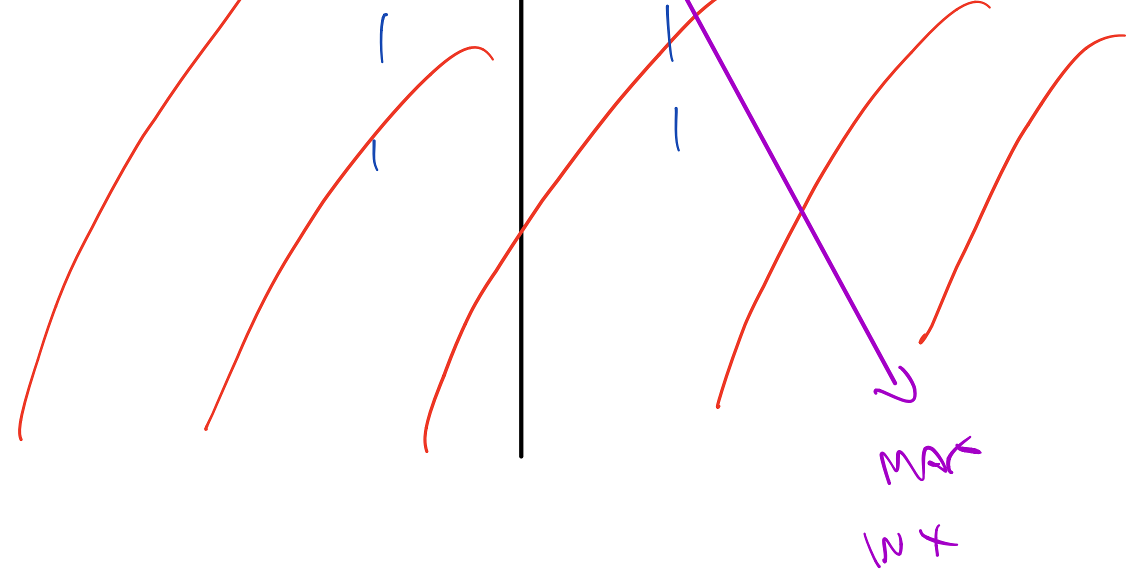
$3x_1^2 + 4x_2 \leq 12 \quad x_2 \geq 0$

$u_1 = \frac{1}{2} u_2$

$-e_1 - e_2 + \frac{1}{2}(x_1)$

$f^* = -x_2 - \frac{1}{2} x_1$





$3x_1^2 + 4x_2 \leq 12$

$4y = -3x^2 + 12$

$y = \dfrac{-3x^2 + 12}{4}$

$O(0,3)$

$-\dfrac{3x^2}{4} + 3 = 0$

$x^2 = 3 \cdot \dfrac{4}{3}$

$x = \pm 2$

$O(2,0) \quad O(-2,0)$

~~Max $(0,3)$~~

~~Min $(-2,0), (2,0)$~~

$$u = f_1 - 9 f_2^2$$

$$\max f_1 = 9x_1^2 + 4x_2^2 - 18x_1 - 16x_2$$

$$\max f_2 = -x_1$$

$$u = 9x_1^2 + 4x_2^2 - 18x_1 - 16x_2 - 9(-x_1)^2$$

$$= \cancel{9x_1^2} + 4x_2^2 - 18x_1 - 16x_2 - \cancel{9x_1^2}$$

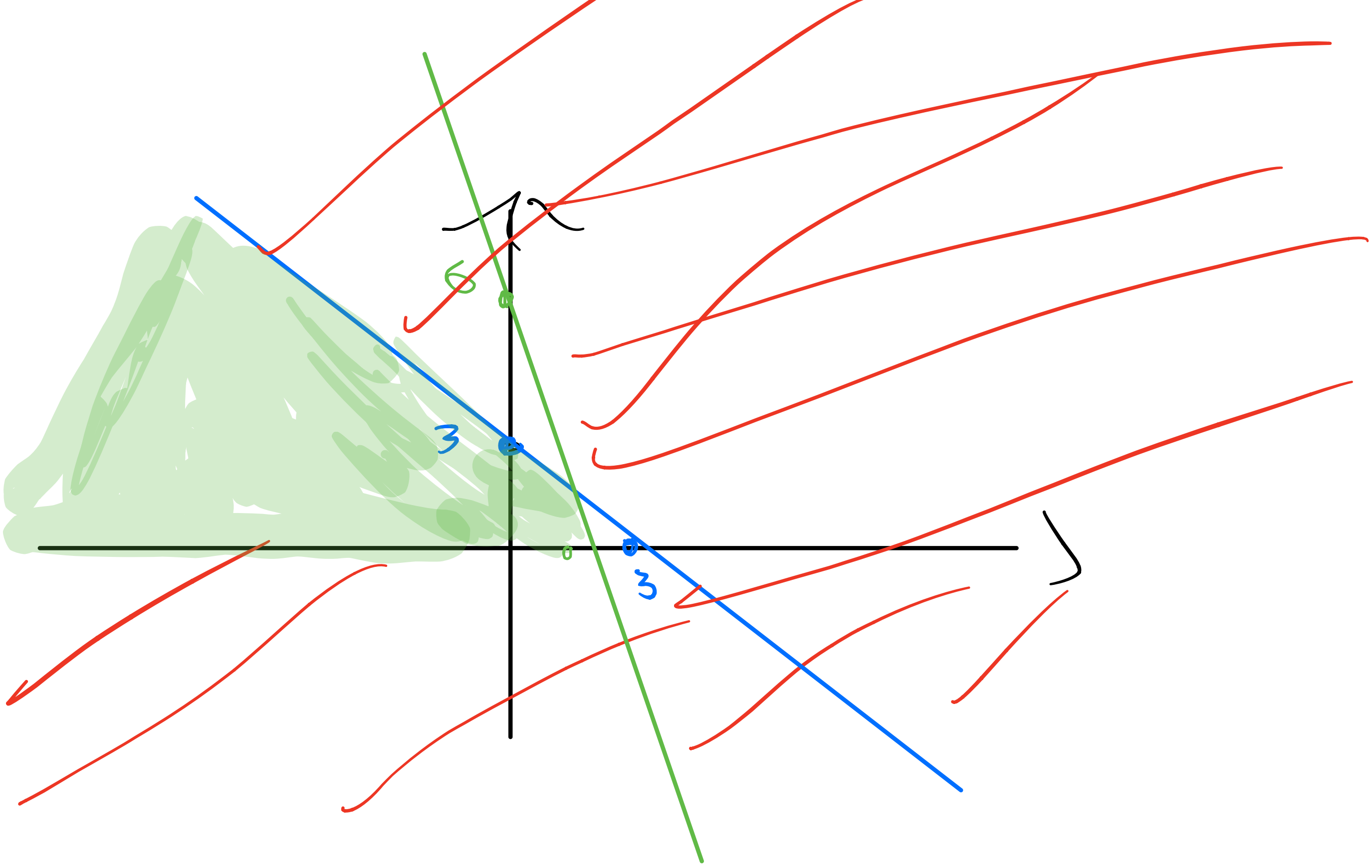
$$f^* = 4x_2^2 - 18x_1 - 16x_2$$

$3x_1 + x_2 = 6$ $\quad y = 6 - 3x_1$ $\quad 0 \to 6$, $2 \to 0$

$$y = \frac{9 - 3x}{3} \qquad y = 3 - x$$

$0 \to 3$

$y \geq 0$ $\quad 3 \to 0$



NON VINCOLATO $\quad x_1 = \infty$

Esplorazione ?

$\min: w_1 f_1 + w_2 f_2$

$w_1 = 0,25 \quad w_2 = 0,75$

$\max f_1 = -x_1 + 2x_2$

$$\max f_1 = 2x_1 - x_2$$

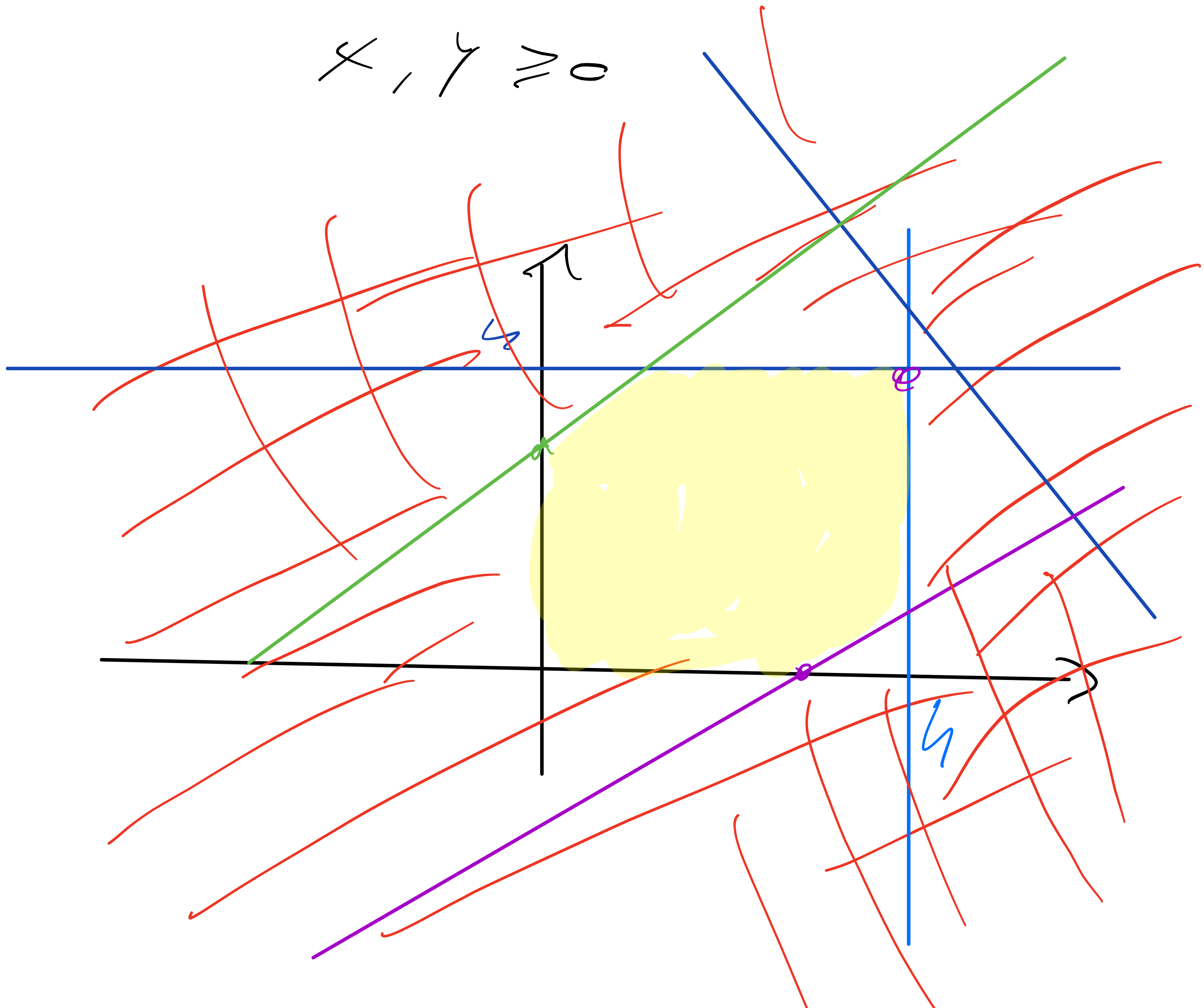
$x_1 + y \leq 7$ $\qquad$ $y = 7 - x$

$-x + y \leq 3$ $\qquad$ $y = 3 + x$

$x - y \leq 3$ $\qquad$ $y = x - 3$

$x, y \leq 4$

$x, y \geq 0$



$$f^{*} = (-x_1 + 2x_2)\frac{1}{4} + \frac{3}{4}(2x_1 - x_2)$$

$$= \frac{-x_1}{4} + \frac{x_2}{2} + \frac{3}{2}x_1 - \frac{3}{4}x_2$$

$$\frac{-x_1 + 2x_2 + 6x_1 - 3x_2}{4} =$$

$$= \frac{5x_1 - x_2}{4} = \frac{5}{4}x_1 - \frac{x_2}{4}$$

$w_3 = 1 - w_1 - w_2$

$w_1 + w_2 \le 1$

$$u' = w_1(\overset{f_1}{-x_1 + 2x_2}) + w_2(\overset{f_2}{2x_1 - x_2}) + (1 - w_1 - w_2)(\overset{f_3}{2x_1 + x_2}) =$$

$= w_1(-3x_1 + x_2) - 2w_2 x_2 + 2x_1 + x_2$

$\frac{1}{4}$ $\frac{3}{4}$

$-\frac{3}{4}x_1 + \frac{1x_2}{4} - \frac{3}{2}x_2 + 2x_1 + x_2$

$$\frac{-3x_1 + x_2 - 6x_2 + 8x_1 + 4x_2}{4}$$

$$u' = \frac{5x_1 - x_2}{4}$$

$\frac{15-4}{4} = \frac{11}{4}$

$\frac{20-3}{4} = \frac{17}{4}$

$\frac{19}{4}$

ES 8.

max $f_1 = x_1 - 3x_2$

max $f_2 = -4x_1 + x_2$

$-2x_1 + 2x_2 \le 7$

$2x_1 + 2x_2 \le 11$

$x_1 \le 4$

$x_1, x_2 \ge 0$

$-2x + 2y = 7$

$2x + 2y = 11$

$y = \frac{7+2x}{2}$

$y = \frac{11-2x}{2}$

$x \le 4$

$1 \to 4,5$

$0 \to 3,5$

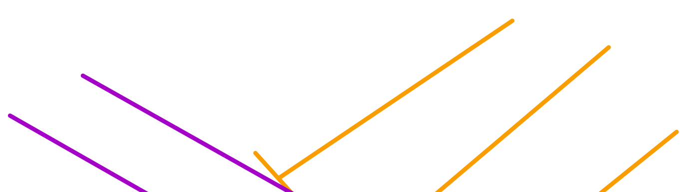
$4 \to \frac{3}{2} = 1,5$

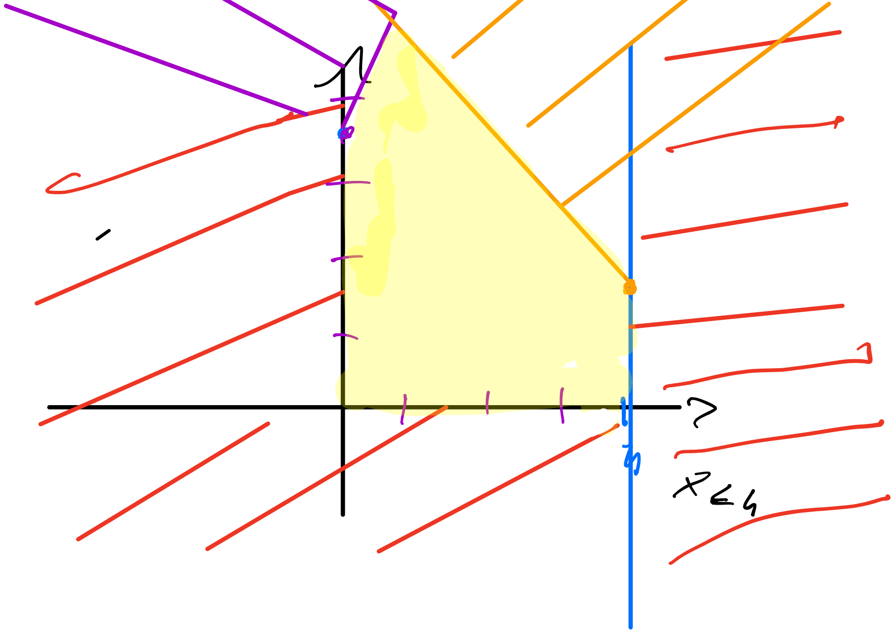
$0 \to \frac{11}{2} = 5,5$

$u = f_1 + f_2$

$u^* = x_1 - 3x_2 + 4(-4x_1 + x_2) =$

$x_1 - 3x_2 - 16x_1 + 4x_2 = x_2 - 15x_1$



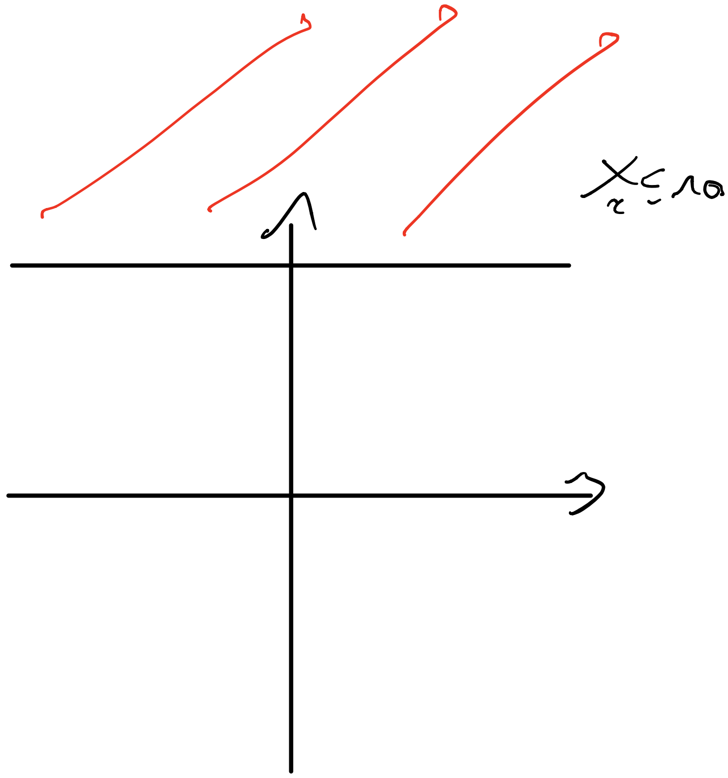


ES S

$$\min \quad f_1(x) = x_1^2 + x_2^2$$

$$\max \quad f_2(x) = x_2$$

$$x_2 \leq 10$$



$f_1(x)$

$$u_1(f_1) \begin{cases} 10 - \dfrac{f_1}{20} & 0 \leq f_1 \leq 200 \\ 0 & f_1 \geq 200 \end{cases}$$

$u_1 = 10 \quad f_1 = 0$

$u_2 \ 10 \quad f_2 = 10$

$U(10, 10) \quad U(0, 10)$

$\boxed{10}$

Indicatori	A	B	C	D	0		Pesi
Costi	90	90	90	1	100	ω_1	$1/3$
Accessibilità	12	13	10	100	37	ω_2	$1/3$
Prestigio	30	1	5	100	10	ω_3	$1/3$

44 34.66 35 67 49

1) $w_1 = 1 - w_2 - w_3 \quad w_2 + w_3 \leq 1$

$$\begin{cases} x + y = 1 \\ 1 - x - y = 0 \end{cases} \quad x = 1 - y$$

11 $\quad f_1 = x^2 - 4x$

$f_2 = -x^2$

$x \geq 0$

$x \leq 3$

$w_1 = w_2$ $\qquad$ $w_1 + w_2 + w_3 = 0$

$$w_1 = \frac{1 - w_3}{2}$$

$$f^* = \frac{1 - w_3}{2}\left(x^2 + 4x\right) - x^2\,\frac{1 - w_3}{2} + w_3$$

$$= 2x - w_3 x$$

$$6 - 3w_3 \geq 2x - w_3 x$$

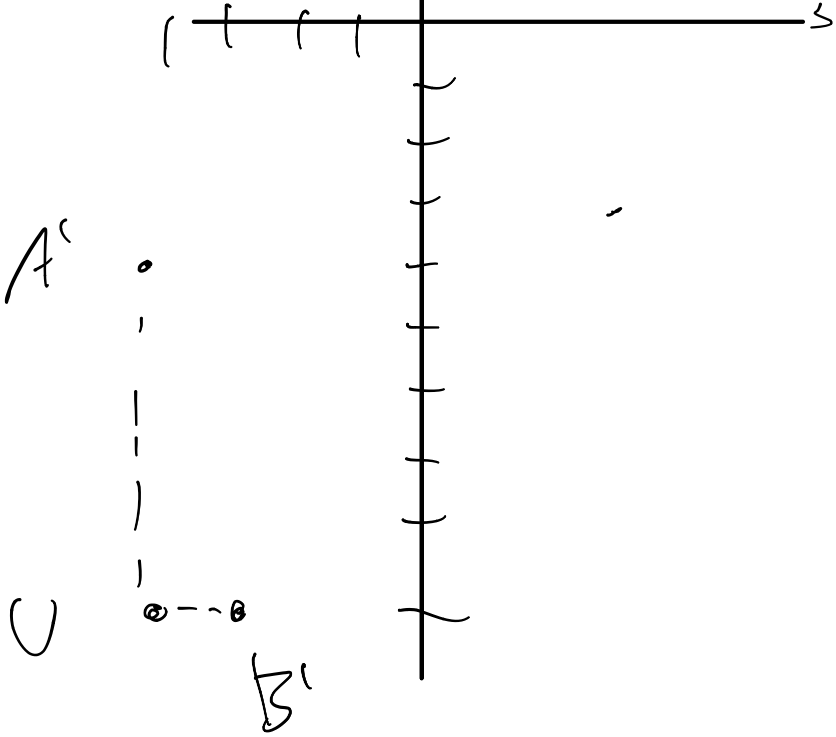
$$6 - 2x \geq (3 - x)\, w_3$$

$$w_3 \leq \frac{6 - 2x}{3 - x} = 2$$

$A'(-4, -4)$ $\qquad$ $B'(-3, -9)$

MIN BETWEEN POINT

$V(-4, -9)$



$$d(A', U) = 5$$

$$d(B', U) = 1$$

$$B' < A'$$

12 U POINTS?

$$\min f_1 = x_1^2 + 4x_2^2 - 2x_1 - 16x_2$$

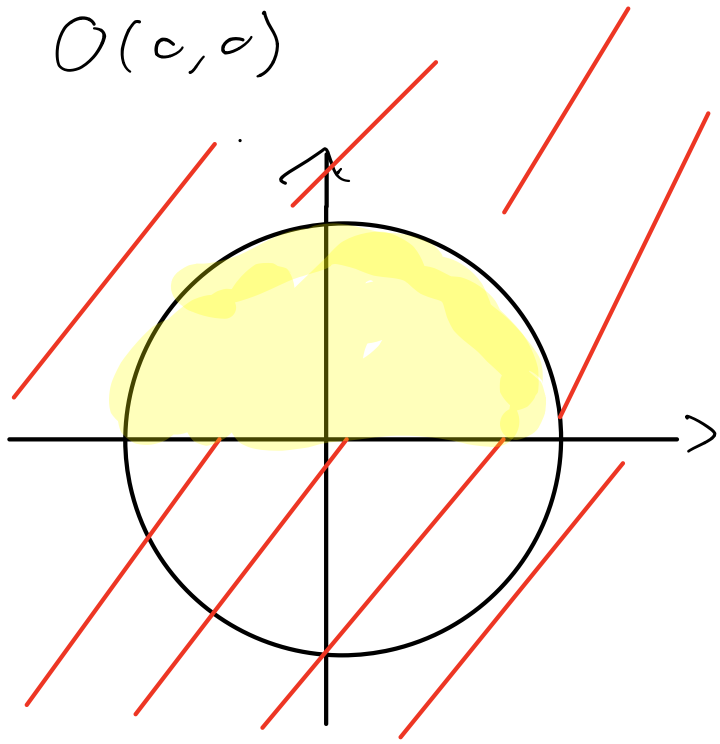
$$\min f_2 = -5x_1 - x_2$$

$$x_1^2 + x_2^2 \leq 8$$

$$x_2 \geq 0$$

$x_1^2 + x_2^2 - 8$

$O(0,0)$



$\sqrt{0+0+8} = \sqrt{8} = 2\sqrt{2}$

$$\begin{cases} x_1^2 + x_2^2 = 8 \\ x_2 = \frac{x_1}{5} \end{cases} \quad \begin{cases} x_1^2 + \frac{x_1^2}{25} = 8 \\ x_2 = \frac{x_1}{5} \end{cases} \quad \begin{cases} x_1^2 = \frac{100}{\sqrt{13}} \\ x_2 = \frac{x_1}{5} \end{cases}$$

$$\begin{cases} x_1 = \frac{10}{\sqrt{13}} \\ x_2 = \frac{2}{\ } \end{cases}$$

$\sqrt{13}$

minimum in ℓ_2

$$\ell_2^* = -\frac{52}{\sqrt{13}}$$

$$x_1(x_1 - 2) \quad 4x_2(x_2 - 4)$$

$$u = 2 \qquad \ell = 16$$

$$-\frac{a}{2} = 1$$

$$-\frac{b}{2} = 2$$

$$x = (1, 2)$$

$$\cancel{1} + 16 \overset{-1}{\cancel{-2}} - 32$$

$$15 - 32 = -17$$

$$\ell_1 = -17$$

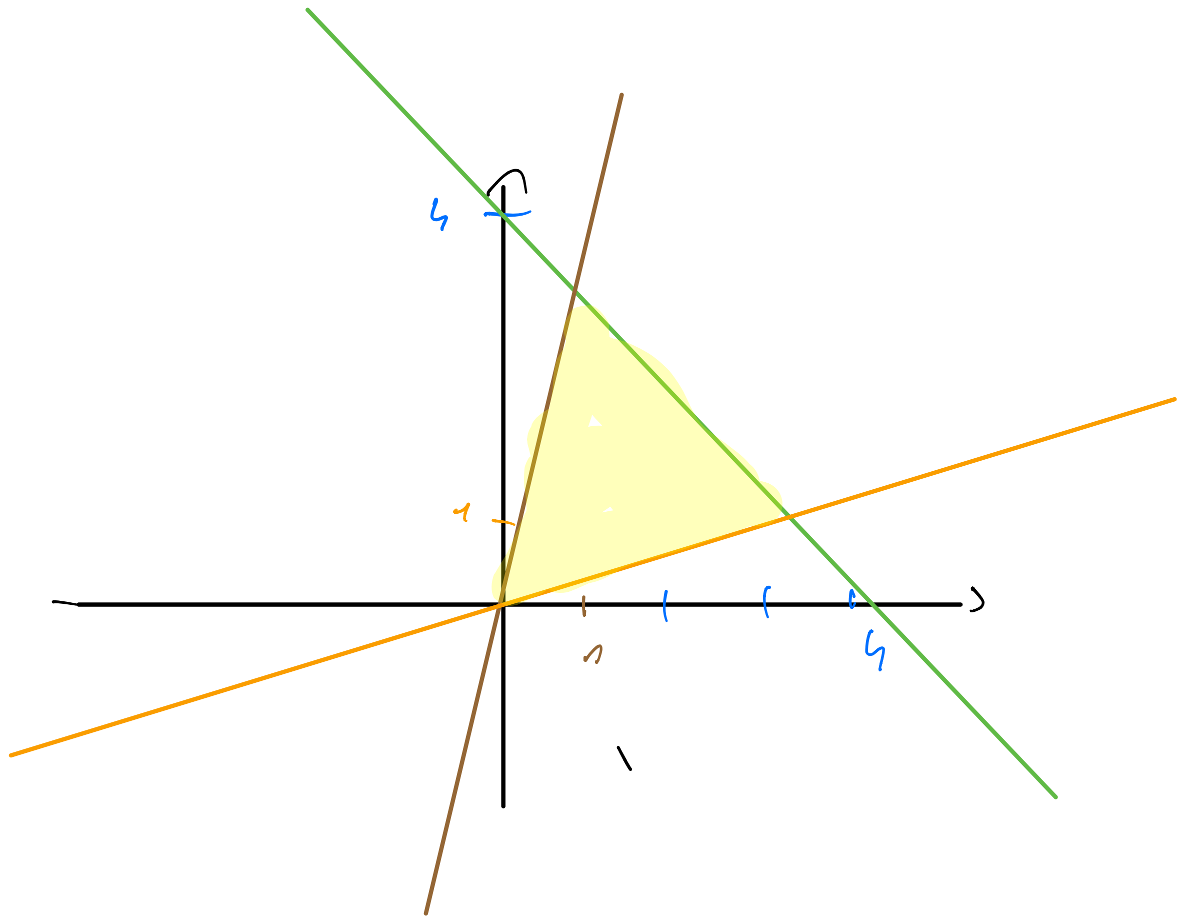
PROVA 02/2018

$\min f(x) = -2x_1 - x_2$

$g_1 = x_1 + x_2 - 4 \leq 0$ $\quad y = 4 - x \quad$ $0 \to 4 \mid 4 - 0$

$g_2(x) = x_2 - 4x_1 \leq 0$ $\quad y = 4x \quad$ $0 \to 0 \mid 1 \to 4$

$g_3 = x_1 - 4x_2 \leq 0$ $\quad y = \frac{x}{4} \quad$ $0 \to 0 \mid 4 \to 1$



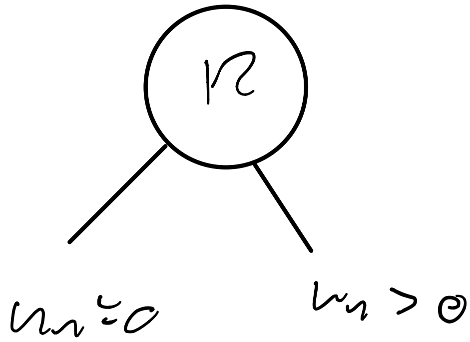
$\nabla g_1 \begin{bmatrix} x_2 & x_1 \end{bmatrix}$ $\quad \nabla g_2 \begin{bmatrix} -4 & 1 \end{bmatrix}$

$$\nabla g_3 = \begin{bmatrix} 1 & -4 \end{bmatrix}$$

$$\nabla f = \begin{bmatrix} -2 & -1 \end{bmatrix}$$

$$L(x) = \begin{vmatrix} -2 \\ -1 \end{vmatrix} + u_1 \begin{vmatrix} x_2 \\ x_1 \end{vmatrix} + u_2 \begin{vmatrix} -4 \\ 1 \end{vmatrix} + u_3 \begin{vmatrix} 1 \\ -4 \end{vmatrix}$$

$$\begin{cases} \begin{vmatrix} -2 \\ -1 \end{vmatrix} + u_1 \begin{vmatrix} x_2 \\ x_1 \end{vmatrix} + u_2 \begin{vmatrix} -4 \\ 1 \end{vmatrix} + u_3 \begin{vmatrix} 1 \\ -4 \end{vmatrix} = 0 \\ u_1 g_1 = u_1(x_1 x_2 - 4) = 0 \\ u_2 g_2 = u_2(x_2 - 4x_1) = 0 \\ u_3 g_3 = u_3(x_1 - 4x_2) = 0 \\ g_1 = x_1 x_2 - 4 \leq 0 \\ g_2 = x_2 - 4x_1 \leq 0 \\ g_3 = x_1 - 4x_2 \leq 0 \\ u_1, u_2, u_3 \geq 0 \end{cases}$$



$$\begin{pmatrix}-2\\-1\end{pmatrix} + u_2 \begin{pmatrix}-4\\1\end{pmatrix} + u_3 \begin{pmatrix}1\\-4\end{pmatrix}$$

$$0(x_1 x_2 - 4) = 0$$

$$u_2 (x_2 - 4 x_1) = 0$$

$$u_3 (x_1 - 4 x_2) = 0$$

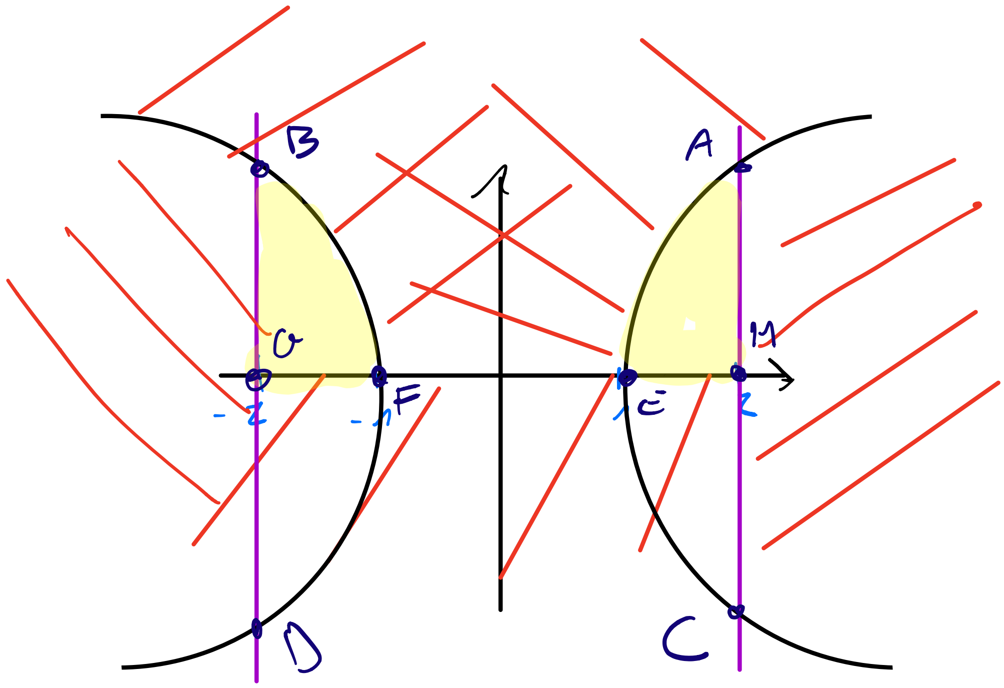
ES 2

$$\min \; f = x_1 + x_2$$

$$g_1 = -x_1^2 + x_2^2 + 1 \le 0 \qquad y^2 \le x^2 + 1 \qquad y = \pm 1$$

$$g_2 = x_1^2 - 4 \le 0 \rightarrow x_1^2 \ge 4 \rightarrow x_1 \ge \pm 2$$

$$g_3 = -x_2 \le 0 \rightarrow x_2 \ge 0$$



$$\nabla g_1 = \begin{bmatrix} -2x_1 & 2x_2 \end{bmatrix} \qquad \nabla g_2 = \begin{bmatrix} 2x_1 & 0 \end{bmatrix}$$

$$\nabla g_3 = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$\delta_1 = \delta_2 = 0$

$$\begin{cases} -x_1^2 + x_2^2 + 1 = 0 \\ x_1^2 - 4 = 0 \end{cases} \qquad \begin{cases} x_2^2 = x_1^2 - 1 \quad \begin{array}{l} \sqrt{3} \\ -\sqrt{3} \end{array} \\ x_1 = \pm 2 \end{cases}$$

- $A(2, \sqrt{3})$

$$\delta_1 \begin{bmatrix} -4 & \sqrt{3} \end{bmatrix} \qquad \delta_2 \begin{bmatrix} 4 & 0 \end{bmatrix}$$

LIN. IND

- $B(-2, \sqrt{3})$

$$\begin{bmatrix} 4 & 2\sqrt{3} \end{bmatrix} \qquad \begin{bmatrix} -4 & 0 \end{bmatrix}$$

- $C(2, -\sqrt{3})$

LIN. IND

- $D(-2, -\sqrt{3})$

LIN IND

$g_1 = g_3 = 0$

$$\begin{cases} -x_1^2 - x_2^2 + 1 = 0 \\ x_2 = 0 \end{cases} \qquad \begin{cases} +x_1^2 = +1 \\ x_2 = 0 \end{cases} \quad \begin{matrix} 1 \\ -1 \end{matrix}$$

$= (1, 0)$

$$\begin{bmatrix} -2 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$F(-1, 0)$

$$\begin{bmatrix} 2 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$\delta_2 = \delta_3 = 0$$

$$\begin{cases} x_1^2 - 4 = 0 \\ x_2 = 0 \end{cases} \quad \begin{cases} x_1 = \begin{matrix} 2 \\ -2 \end{matrix} \\ x = 0 \end{cases}$$

$$O(2, 0) \qquad M(-2, 0)$$

$$\begin{bmatrix} 4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \end{bmatrix}$$

$$\ell(x) = f(x) + \sum_{1}^{K} \delta_K \cdot u_K + \sum_{1}^{\ell} \lambda_\ell g_\ell$$

$$\ell(x) = x_1 + x_2 + \mu_1 (-x_1^2 + x_2^2 + 1) + u_2 (x_1^2 - 4)$$

$+u_3(-x_2)$

$1 + u_1 - 2x_1 + u_2$

$1 + u_1 2 - u_3 = 0$

$u_1(-x_1^2 + x_2^2 + 1) = 0$

$u_2(x_1 - 4) = 0$

$u_3(-x_2) = 0$

g_1, g_2, g_3

$u_1, u_2\ u_3 \geq 0$

$u_3 g_3 = 0$

$u_3 = 0 \quad g \leq 0$

$1 + u_1 - 2x_1 + x_2 = 0$

$1 + 2u_1 x_2 = 0$

$u_1(-x_1^2 + x_2^2 + 1) = 0$

$u_2(x_1 - 4) = 0$

$(x_1^2 - 4) = 0$

$-x_1^2 + x_2^2 + 1 \leq 0$

$x_1^2 - 4 \leq 0$

$-x_2 \leq 0$

$2(u_1 - u_2)x_1 = d$

$1 + 2u_1 x_2 = 0$

$u_1(-x_1^2 + x_2^2 + 1) = 0$

$u_2(x_1^2 - 4) = 0$

$-x_2^2 + x_2^2 \leq -1$

$x_1^2 \leq 4$

$x_2 \geq 0$

$u_1 \geq 0$

$u_2 \geq 0$

$2 u_1 x_2 = -1$

$x_2 \geq 0 \, , \quad u_1 \geq 0$ No!

$$\boxed{u_3 > 0 \quad \delta_3 = 0}$$

$1 - 2 u_2 x_1 + 2 u_3 x_1 = 0$

$1 + \cancel{2 u_1 x_2} - \overset{1}{u_3} = 0 \longrightarrow u_3 = 1$

$u_1 \left(-x_1^2 + \cancel{x_2^2} + 1 \right) = 0$

$u_2 \left(x_1^2 - 4 \right) = 0$

$\cancel{\overset{1}{u_3} x_2 = 0}$

$-x_1^2 + \cancel{x_2^2} + 1 \leq$

$x_1^2 - 4 \leq 0 \quad x_1^2 \geq 4$

$\underline{x_2 = 0}$

$u_3 > 0 \quad \delta_3 = 0$

P^3 $\quad (u_1 = 0 \qquad g_1 \le 0)$

$-2u_2x_1 = 1$ $\qquad u_2 \ge 0$

$u_2(x_1^2 - 4) = 0$ $\qquad x_1 \ge 1$

$u_3 = 1$

$+x_1^2 \ge +1$ $\qquad H(-2, 0)$

$x_1^2 \le 4$

$u_2 \ge 0$

$\mu^1 = (0, \frac{1}{4}, 1)$

P^4 $\quad u_1 > 0 \qquad g_1 = 0$

$1 - 2u_2x_1 + 2u_3x_1 = 0$

$u_3 = 1$

$u_1(-x_1^2 + \quad 1) = 0$

$u_2 \, (x_1^2 - u) = 0$

$\longrightarrow \quad u_2 = 0$

$-x_1^2 + \quad 1 = 0 \longrightarrow x_1^2 = 1$

$x_1^2 - u \leq 0 \quad x_1^2 \geq u$

$x_2 = 0$

$u_3 > 0 \quad \delta_3 = 0$

$E\,(1, 0) \longrightarrow f(E) = 1$

$H\,(-2, 0) \longrightarrow f(H) = -2$

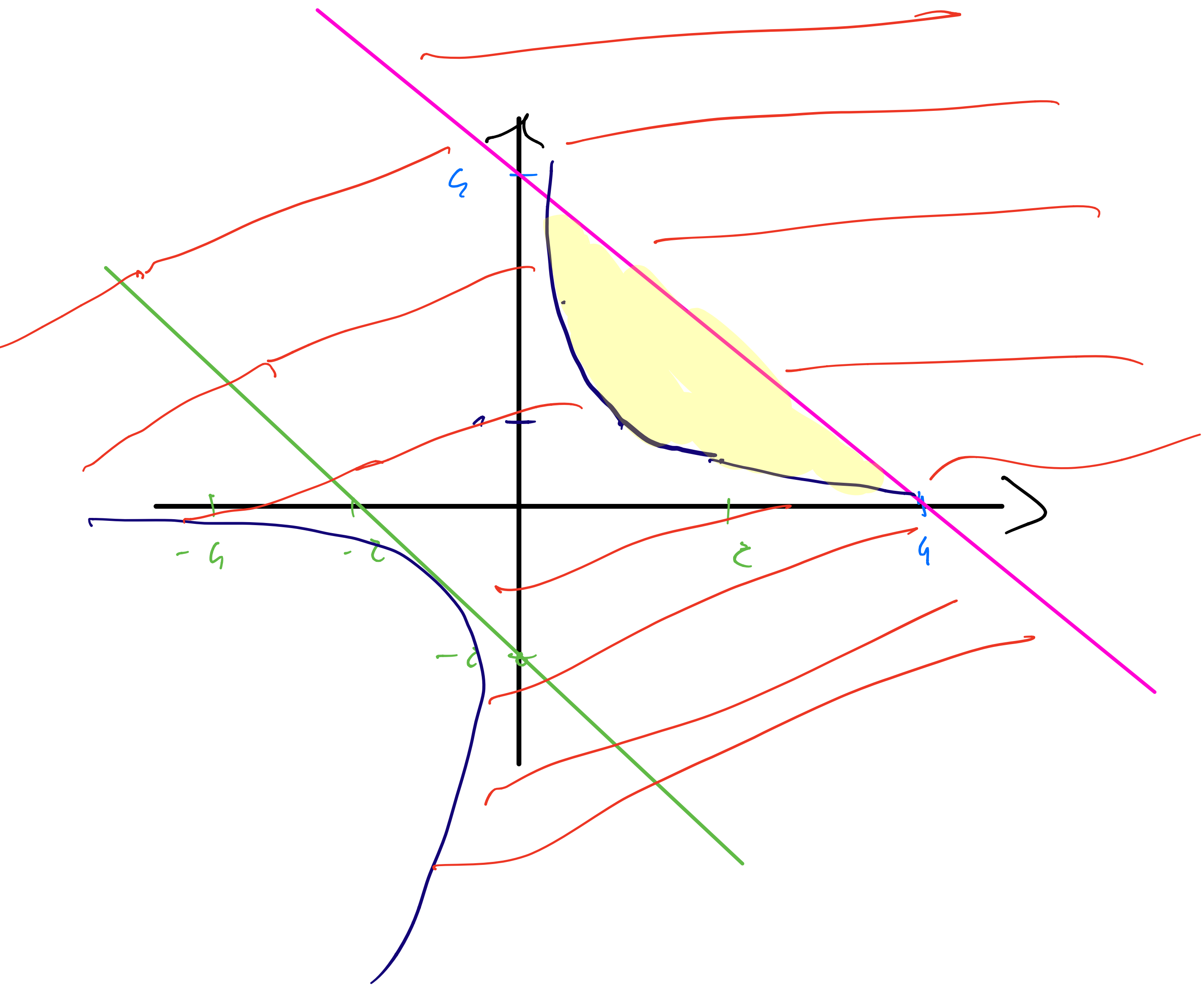
ESAME APRILE 2017

$$\min f(x) = x_1^2 + x_2^2$$

$g_1 = 1 - x_1 x_2 \leq 0 \qquad + y = \frac{+1}{x}$

$g_2 = x_1 + x_2 - 4 \leq 0 \qquad y = -x + 4$

$g_3 = -x_1 - x_2 - 2 \leq 0 \qquad y = -x - 2$



$$\nabla g_1 = \begin{bmatrix} -x_2 & -x_1 \end{bmatrix}$$

$$\nabla g_2 \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad \nabla g_3 = \begin{bmatrix} -1 & -1 \end{bmatrix}$$

g_1 e $g_2 = 0$

$$\begin{cases} 1 - x_1 x_2 = 0 \\ x_1 + x_2 - 4 = 0 \end{cases} \qquad \begin{cases} 1 - \left((-x_2 + 4) \cdot x_2\right) = 0 \\ x_1 = -x_2 + 4 \end{cases}$$

$$\begin{cases} 1 - \left(-x^2 + 4x_2\right) = 0 \end{cases} \qquad \begin{cases} x^2 - 4x_2 + 1 = 0 \\ x_1 = -x_2 + 4 \end{cases}$$

$$x_2 = \frac{4 \pm \sqrt{16 - 4}}{2} \; \begin{cases} 2 + \sqrt{3} \\ 2 - \sqrt{3} \end{cases}$$

$$\frac{-2+\sqrt{3}}{1} \quad \frac{-2+\sqrt{3}}{1}$$

$g_2 = g_3 = 0$

$$\begin{cases} -x_1 - x_2 - 2 = 0 \\ x_1 + x_2 - 4 = 0 \end{cases} \quad \begin{cases} -x_1 + x_1 - 4 - 2 = 0 \\ x_2 = -x_1 + 4 \end{cases}$$

$$\begin{cases} -6 = 0 \\ x_2 = -x_1 + 4 \end{cases}$$

Ne INT

$g_1 = g_3 = 0$

$$\begin{cases} 1 - x_1 x_2 = 0 \\ x_1 + x_2 + 2 = 0 \end{cases} \quad \begin{cases} 1 - ((x_2 - 2) x_2) = 0 \\ x_1 = -x_2 - 2 \end{cases}$$

$$\begin{cases} 1-(-x_2^2-2x_2)=0 \\ \end{cases} \quad \begin{cases} 1-x_2^2+2x_2=0 \\ \end{cases}$$

$$\begin{cases} x_2^2+2x_2+1 = \frac{2\pm\sqrt{4-4}}{2} \quad -1 \\ x_1=-1 \end{cases}$$

$C\,(-1,-1)$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \text{ LI NO}$$

$$\ell(x) = x_1^2+x_2^2+u_1(1-x_1x_2)+u_2(x_1+x_2-4)$$
$$+u_3(-x_1-x_2-2)$$

$$\begin{cases} 2x_1-u_1x_2+u_2-u_3 \\ 2x_2-u_1x_1+u_2-u_3 \\ u_1(1-x_1x_2) \\ u_2(x_1+x_2-4) \end{cases}$$

$u_3(-x_1-x_2-2)$

ESAME 11/6/17

$X \to$ SOLUZIONI POSSIBILI

$\Omega \to$ SCENARI POSSIBILI

$F \to$ IMPATTI POSSIBILI

$f(x,w) : X \times \Omega \to F$ FUNZIONE DI IMPATTO

$D \to$ DECISORI

$\Pi(d) : D \to 2^{F \times F}$ RELAZIONE DI PREFERENZA

3) $\max f_1(x) = x_1 - 2x_2$

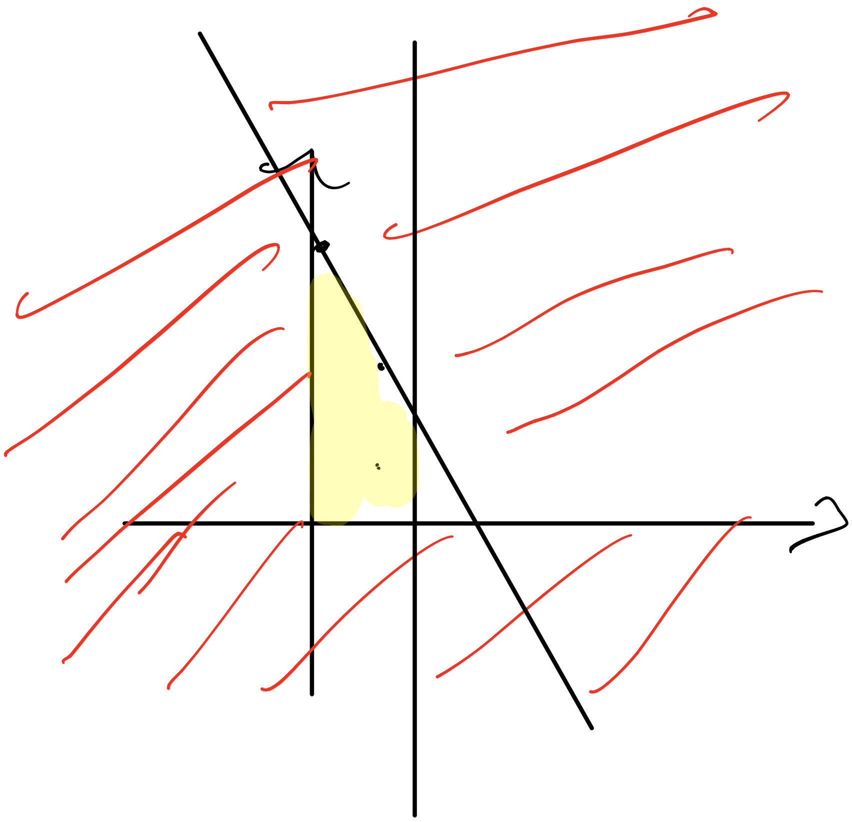
$\max f_2(x) = x_2$

$x_2 \geq 0$

$2x_1 + x_2 \leq 5$ $\qquad x = 5 - 2x_1$

$0 \leq x_1 \leq 2$

$\begin{cases} x_1 = f_1 + 2x_2 \\ x_2 = f_2 \end{cases}$ $\qquad \begin{cases} x_1 = f_1 + 2f_2 \\ x_2 = f_2 \end{cases}$



$$\frac{1}{2}(x_1 - 2x_2) + \frac{x_2}{2}$$

$$\frac{1}{2}x_1 - x_2 + \frac{x_2}{2} \Rightarrow$$

$$\frac{1}{2}x_1 - \frac{1}{2}x_2$$

$$f(0,5) = -\frac{5}{2}$$

$$f(2,0) = 1$$

$$f(2,1) = 1 - \frac{1}{2} = \frac{1}{2}$$

4)

$$\begin{vmatrix} 1 & 3 & \frac{1}{2} & 6 \\ \frac{1}{3} & 1 & \frac{1}{6} & 2 \\ 2 & 6 & 1 & 12 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{12} & 1 \end{vmatrix}$$

PESI > 0

RECIPROCITÀ $x_{lm} = \dfrac{1}{x_{ml}}$

COERENZA $x_{ln} = x_{lm} \, x_{mn}$

$$w_i = \frac{M_i}{\sum_{j=1}^{n} a_{ij}}$$

$$w = \left| \frac{1}{1+3+\frac{1}{2}+6} \right|$$

$$\frac{1}{3} + 1 + 2 \quad \frac{1}{6}$$

$$\frac{2+6+12+1}{6}$$

$$\frac{21}{6}$$

$$\frac{\cancel{6}\ 2}{\cancel{21}\ 7}$$

5

	a_1	a_2	a_3	a_4
u_1	50	40	60	90
u_2	10	30	20	0
	10	30	20	0

$a_3 < a_1$

pessime

a_2 pessime

a_1

$$50\pi(u_1) + 10(1-\pi(u_1))$$

a_2

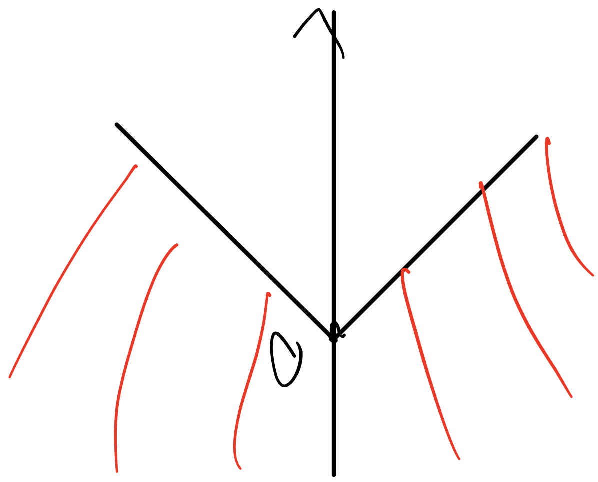
$$40\pi(u_1) + 30(1-\pi(u_1))$$

$$\min f(x) = (x_1 + 1)^2 + \left(x_2 + \frac{1}{2}\right)^2$$

$$g_1(x) = x_1 - x_2 \leq 0$$

$$g_2(x) = x_1 + x_2 \geq 0$$

$$\cancel{g_3(x) = x_2 \geq 0}$$



$x_1^2 + 1 + 2x_1$

$x_2^2 + \frac{1}{4} + x_2$

$$2x_1 + 2 + u_1 + u_2 = 0$$

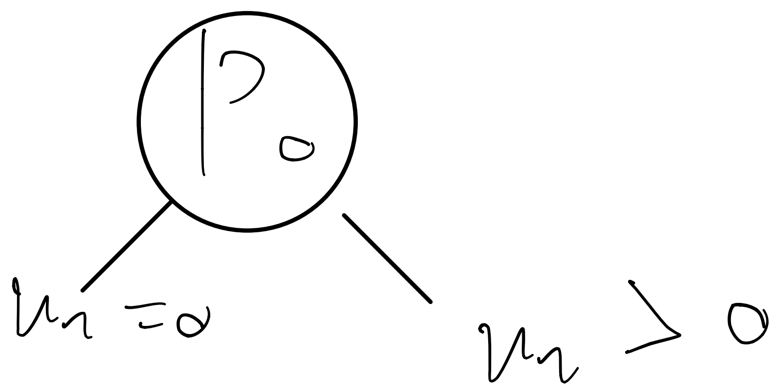
$$2x_2 + 1 - u_2 + u_1 = 0$$

$$u_1(x_1 - x_2) = 0$$

$$u_2(x_1 + x_2) = 0$$

$$x_1 - x_2 \le 0 \longrightarrow x_1 \le x_2$$

$$x_1 + x_2 \ge 0 \longrightarrow x_1 \ge -x_2$$



$u_1 = 0$

$$2x_1 + 2 + u_2 = 0$$

$$2x_2 + 1 - u_2 = 0$$

$$u_2(x_1 + x_2) = 0$$

$$x_1 \leq x_2$$

$$x_1 \geq -x_2$$

$$x_1 = \frac{u_2 - 2}{2}$$

$$x_2 = \frac{u_2 - 1}{}$$

$$2$$

$$-u_2 - 2 = u_2 - 1$$

$$2u_2 = -1$$

$$u_2 = -\frac{1}{2}$$

$$x_1 = \frac{+\frac{1}{2} - 2}{2}$$

$$x_2 = -\frac{1}{2} - 1$$

$$\frac{\quad}{2}$$

$$x_1 = \frac{\dfrac{1-4}{\cancel{2}}}{\cancel{2}} \cdot \cancel{2} = -3$$

$$x_2 = -1 - 2 = -3$$

$$x_1 \geq x_2 \,?$$

$$-3 \geq 3$$ ne!

$u_0 > 0 \qquad S_1 = 0$

$$2x_1 + 2 + u_1 + u_2 = 0$$

$$2x_2 + 1 - u_2 + u_1 = 0$$

$$u_1(x_1 - x_2) = 0$$

$$u_2(x_1 + x_2) = 0$$

$$x_1 - x_2 = 0 \rightarrow x_1 \leq x_2$$

$$x_1 + x_2 \geq 0 \rightarrow x_1 \geq -x_2$$

$$x_1 - x_2 = 0 \qquad x_1 = x_2$$

$$2x_1 + 2 + u_1 + u_2 = 0$$

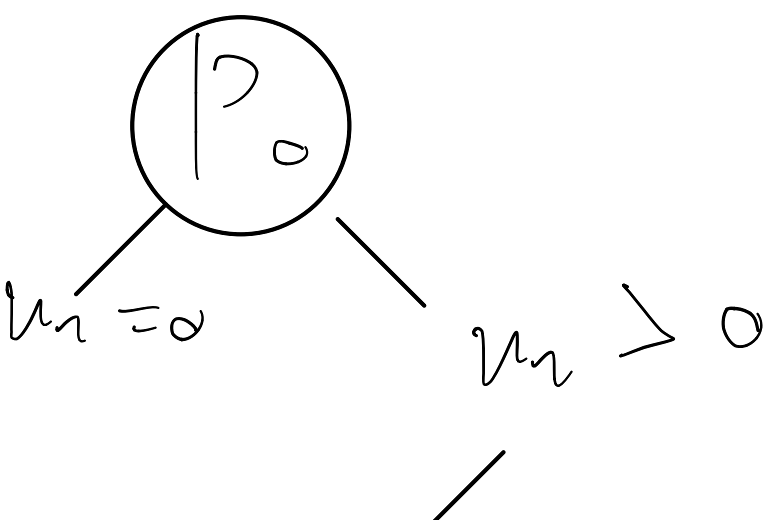
$$2x_1 + 1 - u_2 + u_1 = 0$$

$$u_1(0) = 0$$

$$u_2(2x_1) = 0$$

$$2x_1 \geq 0 \longrightarrow x_1 \geq 0$$

$$x_1 = x_2$$



$u_2 = 0$

$2x_1 + 2 + u_1 + \cancel{u_2} = 0$

$2x_1 + 1 - \cancel{u_2} + u_1 = 0$

$u_1 (0) = 0$

$u_{\cancel{2}} (2x_1) = 0$

$2x_1 \geq 0 \longrightarrow x_1 \geq 0$

$x_1 = x_2$

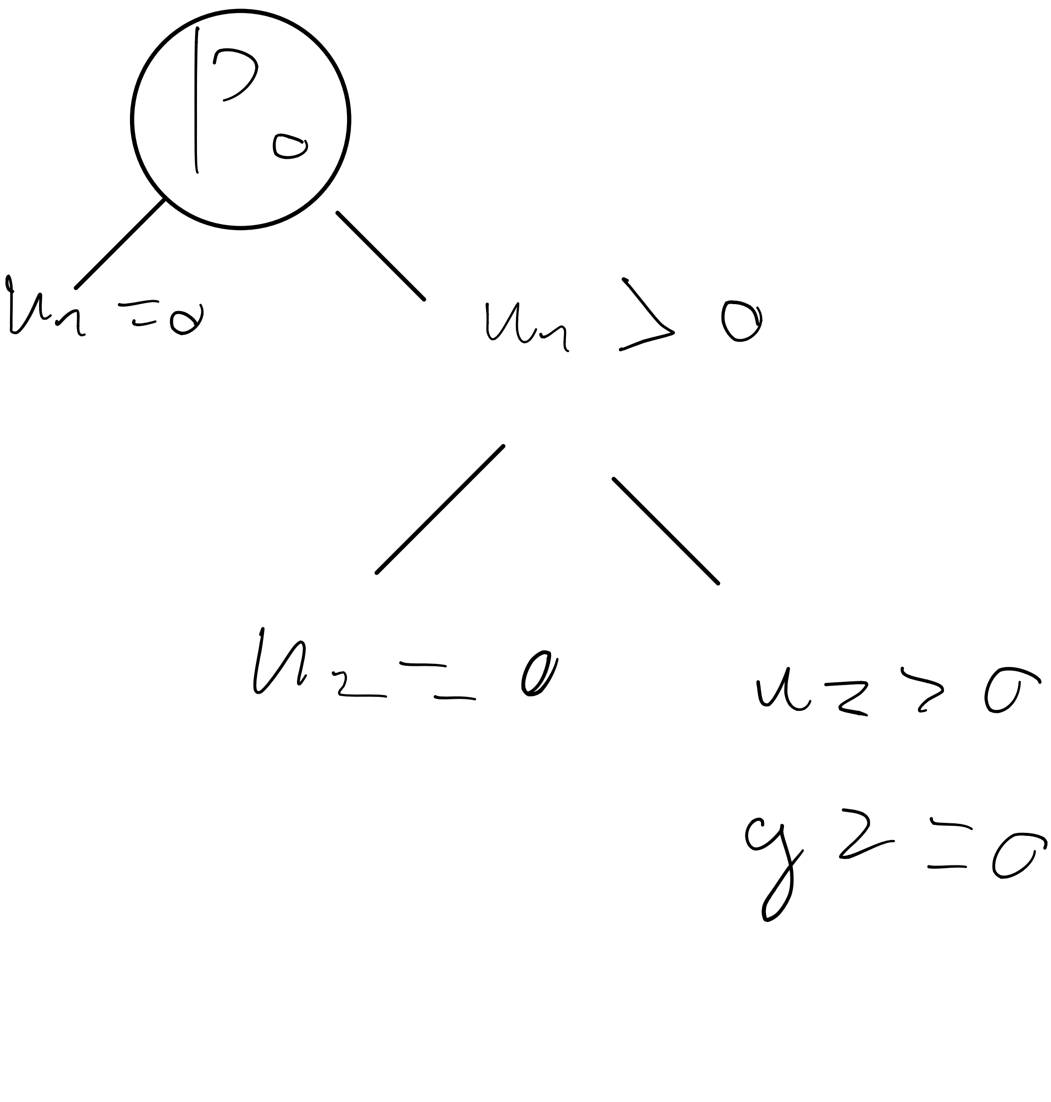
$u_1 = -2x_1 - 2$

$u_1 > 0$

$u_1 = -2x_1 - 1$

$x_1 \geq 0$ no

$x_1 = x_2$



$$2x_1 + 2 + u_1 + u_2 = 0$$

$$2x_1 + 1 - u_2 + u_1 = 0$$

$$u_1 (0) = 0$$

$$u_2 (2x_1) = 0$$

$$x_1 = 0$$

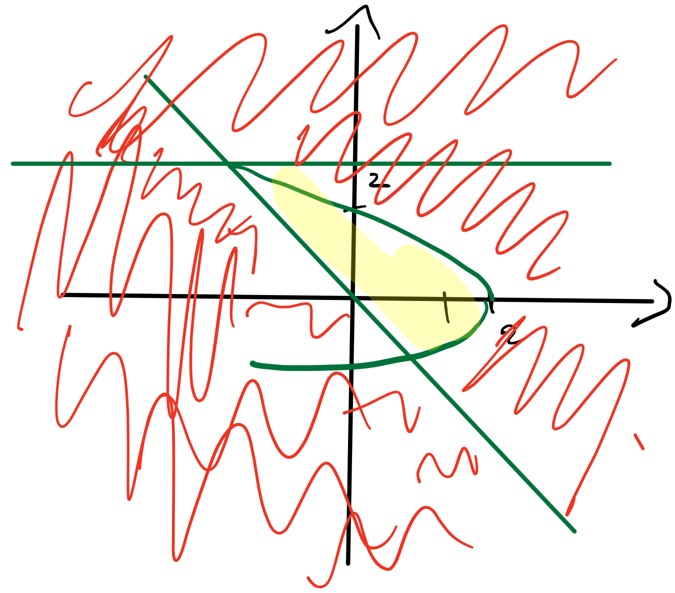
$x_1 = x_2$ $\quad$ $A(0,0)$

$\min f = -x_1 - x_2$

$g_1 = -x_1 - x_2 \le 0$

$g_2 = x_1 - 2 \le 0$

$g_3 = x_1 + x_2^2 - 2 \le 0$



$+y \ge -x_1$

$x_1 \le 2$

$x_2^2 + x_1 - 2 \le 0$

$y^2 \le 2 \quad y \le \pm\sqrt{2}$

$x \le 2$

$(0, \sqrt{2})$

$(0, -\sqrt{2})$

$(2, 0)$

$\nabla g_1 = \begin{bmatrix} -1 & -1 \end{bmatrix} \quad \nabla g_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$\nabla g_3 = \begin{bmatrix} 1 & 2x_2 \end{bmatrix}$

g_1, g_2

$$\left| \begin{array}{l} -x_1 - x_2 = 0 \\ x_1 - 2 = 0 \end{array} \right| \begin{array}{l} -x_2 = 2 \\ x_1 = 2 \end{array} \left| \begin{array}{l} x_2 = -2 \\ x_1 = 2 \end{array} \right.$$

g_1, g_3

$$\left| \begin{array}{l} -x_1 - x_2 = 0 \\ x_2^2 + x_1 - 2 = 0 \end{array} \right| \begin{array}{l} x_1 = -x_2 \\ x_2^2 - x_2 - 2 = 0 \end{array}$$

$$x_2 = \frac{1 \pm \sqrt{1+8}}{2} \begin{cases} \frac{1+3}{2} = 2 \\ \frac{1-3}{2} = -1 \end{cases}$$

$A(-2, 2)$ $B(1, -1)$

g_3, g_2

$$\begin{cases} x_2^2 = 0 \\ x_1 = 2 \end{cases}$$

$C(2, 0)$ DOP NUN ZERO

$$\min f = -x_1 - x_2$$

$$g_1 = -x_1 - x_2 \le 0$$

$$g_2 = x_1 - 2 \le 0$$

$$g_3 = x_1 + x_2^2 - 2 \le 0$$

$$l = f(x) + \sum h_i \lambda + \sum u_i g_i$$

$$-x_1 - x_2 + u_1(-x_1 - x_2)$$
$$+ u_2(x_1 - 2)$$
$$+ u_3(x_1 + x_2^2 - 2)$$

$$-1 - u_1 + u_2 + u_3 = 0$$

$$-1 - u_1 + 2x_2 u_3 = 0$$

$$u_1(-x_1 - x_2) = 0$$

$$u_2(x_1 - 2) = 0$$

$$u_3(x_1 + x_2^2 - 2) = 0$$

$$-x_1 - x_2 \leq 0$$

$$x_1 - 2 \leq 0$$

$$x_1 + x_2^2 - 2 \leq 0$$

P_0

$u_2 \leq 0$ $\qquad$ $u_2 > 0$

$$-1 - u_1 + u_3 = 0$$

$$-1 - u_1 + 2x_2 u_3 = 0$$

$$u_1(-x_1 - x_2) = 0$$

$$u_3(x_1 + x_2^2 - 2) = 0$$

$$-x_1 - x_2 \leq 0 \rightarrow x_2 \geq -x_1$$

$$x_1 \leq 2$$

$$x_1 + x_2^2 - 2 \leq 0$$

$$u_3 = u_1 + 1 \longrightarrow u_1 = u_3 - 1$$

$$u_1(-1 + 2x_2) + 2x_2 = 1$$

$$u_1(-x_1 - x_2) = 0$$

$$u_3(x_1 + x_2^2 - 2) = 0$$

$x_2 \geq -x_1$

$x_1 \leq 2$

$x_2^2 + x_1 - 2 \leq 0$

$$x_2 = \frac{1 - u_1(-1 + 2x_2)}{2}$$

Γ_0

$u_2 = 0$ $\quad$ $u_2 > 0$

$u_1 = 0$ $\quad$ $u_1 > 0$

$u_1 = 0$

$u_3 = \cancel{u_1} + 1 \rightarrow \cancel{u_1} = u_3 - 1$

$\cancel{u_1(-1+2x_2)} + 2x_2 = 1$

$\cancel{u_1(-x_1 - x_2) = 0}$

$u_3(x_1 + x_2^2 - 2) = 0$

$x_2 \geq -x_1$

$x_1 \leq 2$

$x_2^2 + x_1 - 2 \leq 0$

$u_3 = 1$

$x_2 = \frac{1}{2}$

$x_1 + \frac{1}{4} - 2 = 0$

$x_1 = 2 - \frac{1}{4}$ $\quad x_1 = \frac{8-1}{4}$

$x_1 = \frac{7}{4}$ $\quad x_2 = \frac{1}{2}$

Γ_0

$u_2 = 0$ $\qquad u_2 > 0$

$u_1 = 0$ $\qquad u_1 > 0$

$\left(\frac{7}{4}, \frac{1}{2}\right)$ $\qquad$ imp

$u_3 = u_1 + 1 \rightarrow u_1 = u_3 - 1$

$u_1(-1 + 2x_2) + 2x_2 = 1$

$u_1(-x_1 - x_2) = 0$

$(-x_1 - x_2) = 0$

$u_3 (x_1 + x_2^2 - 2) = 0$

$x_2 \geq -x_1$

$x_1 \leq 2$

$x_2^2 + x_1 - 2 \leq 0$

$u_1 > 0$

$-x_1 - x_2 = 0 \quad x_1 = -x_2$

$u_1 (-1 + 2x_2) + 2x_2 = 1$

$x_2^2 - x_2 - 2 \leq 0$

$-u_1 + 2x_2 u_1 + 2x_2 = 1$

$$2x_2(u_1+2) - u_1 = 1$$

$$x_2 = \frac{1+u_1}{2(u_1+2)}$$

$$- \frac{1+u_1}{2(u_1+2)} \geq + x_1$$

$$x_1 \leq 2$$

$$- 1+u_1 \geq 4(u_1+2)$$

$$-1+u_1 \geq 4u_1+8$$

$$3u_1 \leq -9 \qquad u_1 \leq -3$$

Nope!

≥ 0

$u_2 = 0$	$u_2 > 0$
$u_1 = 0$: $\left(\frac{7}{4}, \frac{1}{2}\right)$; $u_1 > 0$: imp	$(2, 0)$

$$
\left|\begin{aligned}
&-1 - u_1 + u_2 + u_3 = 0 \\
&-1 - u_1 + 2x_2 u_3 = 0 \\
&u_1(-x_1 - x_2) = 0 \\
&u_2(x_1 - 2) = 0 \\
&u_3(x_1 + x_2^2 - 2) = 0 \\
&-x_1 - x_2 \leq 0 \\
&x_1 - 2 \leq 0 \\
&x_1 + x_2^2 - 2 \leq 0
\end{aligned}\right.
$$

$u_2 > 0 \ , \ s_2 = 0$

$-1 - u_1 + u_2 + u_3 = 0$

$-1 - u_1 + 2x_2 u_3 = 0$

$u_1(-x_1 - x_2) = 0$

$u_2(x_1 - 2) = 0$

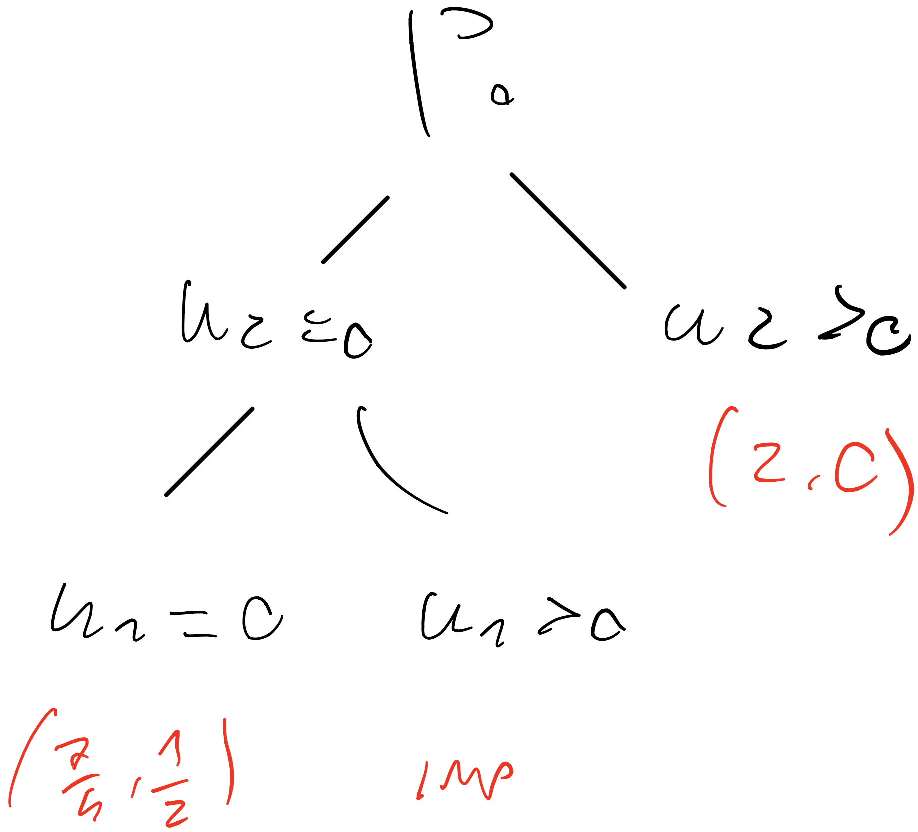
$u_3(x_1 + x_2^2 - 2) = 0$

$-x_1 - x_2 \leq 0 \quad +x_2 \geq -2$

$x_1 - 2 = 0 \quad \text{———} \quad x_1 = 2$

$x_2^2 \leq 0 \qquad x_2 = 0$

$(2, 0)$



$$\frac{7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1} \cdot \cancel{3} \cdot \cancel{2}}$$