Università degli Studi di Milano Corso di studi in Informatica/Matematica

Decision methods and models
(Prof. Roberto Cordone)
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Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Briefly define the concepts of *scenario* and *scenario element* in a decision problem, explaining their role in the decision process.

Briefly define the order relations most commonly assumed in a decision problem.

Briefly define the concept of *stakeholder* in a decision problem.

What is the difference (if any) between a *decision-maker* and a *stakeholder* in a decision problem?

Exercise 2 - Given the following mathematical programming problem:

$$\begin{aligned} &\min f(x) = x_2 \\ &x_1^2 + (x_2 - 1)^2 \leq 1 \\ &x_1 - x_2 \leq -2 \end{aligned}$$

a) represent it graphically;

b) determine the nonregular points (if any exist);

c) determine the candidate points according to Karush-Kuhn-Tucker's conditions, and in particular the global minimum point(s).

Exercise 3 - Briefly explain the relation between *additivity of the utility function* and *mutual preferential independence*.

Given the following biobjective problem:

$$\begin{aligned} &\max f_1(x) = x_1 - 2x_2 \\ &\max f_2(x) = x_2 \\ &\text{with} x \in X = \{x : 2x_1 + x_2 \leq 5;\ 0 \leq x_1 \leq 2;\ x_2 \geq 0\} \end{aligned}$$

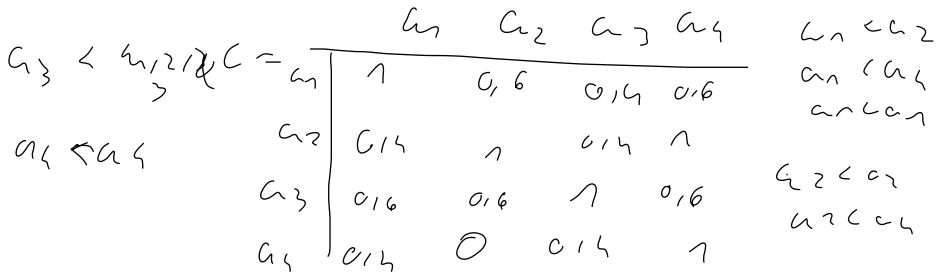
a) represent it graphically both in the decision-variable space $x_1 - x_2$ and in the objective space $f_1 - f_2$;

b) determine the Paretian region with the ϵ-constraint method, replacing the second objective with a constraint.

Exercise 4 - Briefly define the *outranking relation* used by the *ELECTRE methods* and explain how it differs from a classical preference relation.

Given the following evaluation matrix, whose values represent **benefits**

u_{fa}	a_1	a_2	a_3	a_4
f_1	0.90	0.98	0.50	0.94
f_2	0.10	0.09	0.50	0.08



build the outranking relation based on the thresholds $\epsilon_1 = \epsilon_2 = 0.03$ and the corresponding *kernel*.

Refine the relation with the *concordance* criterium $w^{+}_{ff'} + w^{=}_{ff'} \geq \alpha_c = 0.5$ with weight vector $w = [\ 0.4\ 0.6\ ]'$.

Exercise 5 - Given the following evaluation matrix, whose values represent **benefits**

$u_{\omega a}$	a_1	a_2	a_3	a_4
ω_1	70	50	100	60
ω_2	30	40	10	20

a) indicate the dominated alternatives and (if any exist) which alternatives dominate them;

b) choose an alternative with the *worst-case criterium*;

c) show the alternatives chosen with the *Hurwicz criterium* as the *pessimism coefficient* α varies.

Exercise 6 - The following tables provide the benefits $f(x, \omega)$ associated to all possible configurations of a decision problem in conditions of risk, and the conjoint probabilities $\pi(y, \omega)$ of the outcomes y of a random experiment and the scenarios ω.

$f(x,\omega)$	ω_1	ω_2	ω_3
x_1	10	55	25
x_2	65	15	5

$\pi(y,\omega)$	ω_1	ω_2	ω_3
y_1	0.30	0.05	0.15
y_2	0.06	0.40	0.04

Solve the problem with a decision tree adopting the *expected value criterium* without performing the random experiment.

Solve it again performing the random experiment and determine the value of the information it provides.

Exercise 7 - Given the following *payoff matrix* for a two-player game:

	a	b	c
a	3,8	6,9	2,10
b	4,5	9,2	8,6
c	1,7	5,0	10,2

determinine the dominated strategies and the Nash equilibria (if any exist).

Briefly describe the concept of *extended form* of a game.

Briefly define the concept of *prisoner's dilemma* game.

Exercise 8 - Briefly describe the *plurality system* to aggregate preferences and its disadvantages.

Briefly define the concept of *dictator* according to Arrow's theory.

Briefly define the concept of *dependence from irrelevant alternatives* and explain why it is considered a problem for group decisions.

Briefly describe the *sovereignty axiom* in Arrow's theory.