Lezione 1 – 23/09/2019

venerdì 1 novembre 2019 19:40

Exam

Open-ended question and computer out to comment
+ Mini project

BOOKS
time series analysis j.D Hamilton

Introduction to time series and forecasting Brokwell Peter J

Stanza 4 lun 12,30 - 14.30

Time series

Study data observed over time using statistic technic.
What happen today depend on what happened yesterday.
Block of observation between two days:
Y1, y2 ... This are all observed
t = T, t = 1-Y1, y2 , ... Yt, Yt+1, ... YT

We can observe that:
• YT depends on Ys if s < t
NO OBSERVATIONS ARE MISSING
• Yt not depends on Ys if s > t
What happen today does not depends on what happens tomorrow

Vector {y1, y2 ..n, Yt, Yt+1, ...YT} '
Is a time series
Observation depends on the past but not in the futures

It's a random vector that contains random variables with mean, variance, standard deviation and correlation
μ -> mean
σ -> standard deviation
γ -> variance
p -> correlation

OPERATORS

lag operator : L
$L^{-1} Y_t = Y_{t+1}$

First difference operator: Δ = 1 – L
$\Delta Y_t = Y_t - Y_{t-1}$
$\Delta^2 Y_t = (1 - L)^2 Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$

Inverse of lagging to go in the future
$L^{-1} Y_t = Y_{t+1}$ → $Y_t = L\, Y_{t+1}$
↓
$\frac{Yt}{L} = Y_{t+1}$

Distribution of all coin tossing is the same and are independent. At 100° toss we have ½ probability. Y → Tossing a coin
Vector is one sample of 1 observation only.

Stationary and Ergocity

One single realisation $\{ Y_{1, \dots,} Y_T \}'$
$\{Y_t\}_t{}^{\infty} = -\infty$
There are t components in this observation

- IDENTICAL → All moments are the same
  There is not much heterogeneity over time

- INDIPENDENT → All events are independent
  There is not much dependence over time

If there is a low dependency maybe it would work

Ex
Y_t, ~~Y_2~~, ..., ~~Y_{t-1}~~, Y_t ,... Y_T
We can delete Y_2, Y_{t-1} to remove dependency.
Y_t depended on Y_{t-1}, but not on Y_{t-2} and so on.
With elimination we have smaller number of observation and they are independent

RESTRICT HETEROGENETICITY

Covariance stationary
$E(Y_t) = \mu \ \forall\ t$
Weak stationarity

Strict stationarity → require all distribution are the same
So, distribution doesn't change

SUFFICIENT CONDITION FOR STATIONARITY

White noises

I can obtain stationarity process using

$$Yt = \mu + \sum_{j=0}^{\infty} \varphi_j \ \varepsilon_t - j$$

E = 0 Var = **σ²**
Mean is the same

IF. $\sum_{j=0}^{\infty} \varphi_j < \infty$ ⇒ After some time we got 0. Like integral, exist when they push to ∞

ε_t is white noise, then Y_t is stationary
What I'm trying to do is breaking dependence

So two condition:
1) We go to 0 very quickly
2) Independence

MIXING → one theoretical restriction that makes dependence "go away"

Example
$Y_t = \mu + Yt$
WHERE μ → outcome of coin toss of 1 €
Yt → outcome of coin the 1 £ coin

$\frac{1}{2}\mu + \frac{1}{2}Yt$ → Expected value
All dependent to 1 € coin
Sample average

$E(X_t) = 1$ but $E(Y_t)$
When M = 1:
$E(X_t \mid M = 1) = 3/2$
$E(X_t \mid \mu = 0) = ½$

Ergodicity (Heuristic)

When I have an identical process and get average. When I take sample average, I take population average. The moments are not too strange.

Examples
1. Skipped for now

2. **MA1 Model**

   $\{\varepsilon_t\}_t^{\infty} = -\infty$ WHERE $E(\varepsilon_t) = 0$, $VAR(\varepsilon_t) = \sigma^2$

   $\{Y_t\}_t^{\infty} = -\infty$

   $Y_t = \mu + \varepsilon_t + \theta \varepsilon_{t-1}$
   Its ergodicity doesn't depend on what happened before

   $$Y_t = \frac{c}{1 - \phi} + \sum_{j=0}^{\infty} \varphi_j \ \varepsilon_t - j$$
   Go to 0 quickly … Ergotic