2019

Term Structure Of Interest Rates

TIME SERIES ECONOMETRICS PROJECT

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1. Introduction

The aim of the project is to analyse the dataset that contains US dollar LIBOR interbank interest rates. It contains two type of maturity observations of the period from 1961 to 2008: monthly short-term and yearly long-term.
The study is focused on searching for relation between the long-term and short-term maturity of interest rates, checking for the cointegration, the dependence and the response between the two variables.

We choose to analyse the rates with 3 months maturity (M3) and 3 years maturity (Y3).

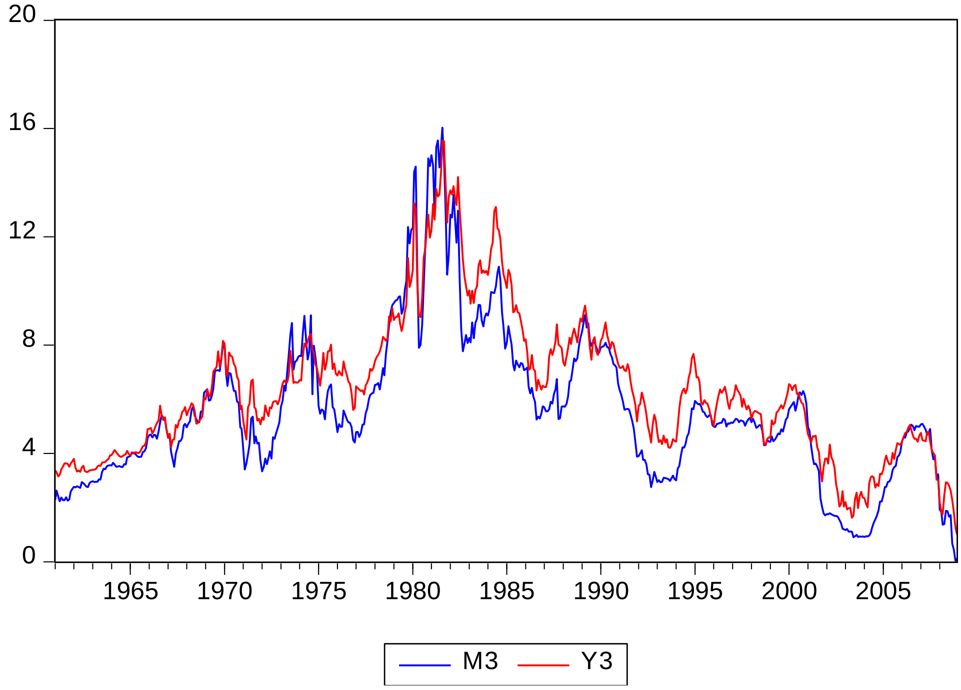

Figure 1: Plot of M3 and Y3

From the plot (Figure 1) is possible to see that the series does not have a trend because does not consistently increase or decrease during time. Also, is possible to deduce that the short-term maturity rates follow the long-term maturity (Y3). In fact, the long-term rates anticipate the short ones. When Y3 increase, M3 follows lags later.

2. Unit root test

With the unit root test is possible to test whether a time series variable is I(1) and possesses a unit root. On EViews, we tested the series with the Augmented Dickey-Fuller test. The parameters specified are no trend and intercept (since from the plot is possible to see that the series has no trend) and lags using Schwarz information criterion.

2.1 Unit root test for short-term rates

For the short-term unit root (M3):

Null Hypothesis: M3 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.262989	0.1846
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(M3)
Method: Least Squares
Date: 12/04/19 Time: 20:32
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
M3(-1)	-0.017221	0.007610	-2.262989	0.0240
D(M3(-1))	0.145014	0.041488	3.495341	0.0005
C	0.092359	0.047519	1.943634	0.0524

R-squared	0.027038	Mean dependent var	-0.004384
Adjusted R-squared	0.023630	S.D. dependent var	0.514612
S.E. of regression	0.508495	Akaike info criterion	1.490491
Sum squared resid	147.6420	Schwarz criterion	1.513240
Log likelihood	-424.7709	Hannan-Quinn criter.	1.499364
F-statistic	7.933838	Durbin-Watson stat	1.986213
Prob(F-statistic)	0.000399		

Figure 2: Unit root test for M3 outputs

The p-value is 0.1846 (Figure 2) which is greater than the significant 0.05. Thus, it is not enough evidence to reject the null hypothesis of having a unit root in short-term interest rate. We can conclude that M3 is I(1).

2.2 Unit root test for long-term rates

For the long-term unit root (Y3):

Null Hypothesis: Y3 has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.825867	0.3679
Test critical values:	1% level	-3.441513	
	5% level	-2.866356	
	10% level	-2.569395	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y3)
Method: Least Squares
Date: 12/04/19 Time: 20:33
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y3(-1)	-0.011922	0.006529	-1.825867	0.0684
D(Y3(-1))	0.137645	0.041560	3.311985	0.0010
C	0.072317	0.044976	1.607900	0.1084

R-squared	0.022767	Mean dependent var	-0.004030
Adjusted R-squared	0.019344	S.D. dependent var	0.416418
S.E. of regression	0.412371	Akaike info criterion	1.071426
Sum squared resid	97.09848	Schwarz criterion	1.094175
Log likelihood	-304.4994	Hannan-Quinn criter.	1.080300
F-statistic	6.651397	Durbin-Watson stat	1.976272
Prob(F-statistic)	0.001395		

Figure 3: Unit root test for Y3 outputs

Also, for Y3 there is not enough evidence to reject the null hypothesis since the p-value is 0.3679 (Figure 3) which is greater than 0.05. Since there is a unit root for Y3, then is possible to define it as I(1).

2.3 Unit root test for spread

It is possible to apply a unit root test for the Spread in order to define whether is I(1) or I(0). Since spread can be used to derive expectations of future rates dynamics, could be useful to check whether is I(0) or I(1). In fact, we can read in the term spread of today if the market expects short rates to raise or fall in the future and also we can read in the term spread of today if the market expects long rates to raise or fall in the future.
We generate a new time series using the difference between Y3 and M3.
On EViews we specify a new time series SPREAD = Y3 – M3 and generate the plot (Figure 4).

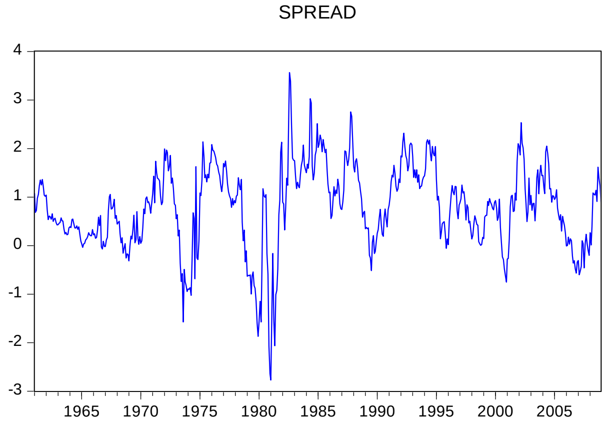

Figure 4: Spread of Y3 and M3

Applying the unit root test to the spread:

Null Hypothesis: SPREAD has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.207022	0.0000
Test critical values:	1% level	-3.441493	
	5% level	-2.866348	
	10% level	-2.569390	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(SPREAD)
Method: Least Squares
Date: 12/09/19 Time: 13:40
Sample (adjusted): 1961M02 2008M12
Included observations: 575 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SPREAD(-1)	-0.090269	0.017336	-5.207022	0.0000
C	0.069840	0.020222	3.453611	0.0006

R-squared	0.045180	Mean dependent var	-0.000335
Adjusted R-squared	0.043514	S.D. dependent var	0.369665
S.E. of regression	0.361533	Akaike info criterion	0.806545
Sum squared resid	74.89457	Schwarz criterion	0.821691
Log likelihood	-229.8817	Hannan-Quinn criter.	0.812452
F-statistic	27.11308	Durbin-Watson stat	1.947463
Prob(F-statistic)	0.000000		

Figure 5: Unit root test for spread

Since the p-value is equal to 0 (Figure 5) we reject the null hypothesis of having a unit root, then spread is I(0).

3 Test for Cointegration

Cointegration test is useful to determine whether two I(1) variables are cointegrated. If a linear combination of two variable is I(0), then we conclude that the variables are cointegrated. For two I(1) variables is possible to apply Engle-Granger test to check for cointegration.

3.1 Engle-Granger cointegration test

On EViews is possible to use the Engle-Granger test (Figure 6).

Date: 12/04/19 Time: 20:36
Series: M3 Y3
Sample: 1961M01 2008M12
Included observations: 576
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C
Automatic lags specification based on Schwarz criterion (maxlag=18)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
M3	-5.219229	0.0001	-52.05442	0.0000
Y3	-5.064235	0.0001	-49.33234	0.0001

*MacKinnon (1996) p-values.

Intermediate Results:

	M3	Y3
Rho - 1	-0.090529	-0.085795
Rho S.E.	0.017345	0.016941
Residual variance	0.130785	0.111171
Long-run residual variance	0.130785	0.111171
Number of lags	0	0
Number of observations	575	575
Number of stochastic trends**	2	2

**Number of stochastic trends in asymptotic distribution

Figure 6: Engle-Granger test outputs

We choose the long-term as a dependent variable for the test. We reject the null hypothesis (p-value < 0.05) that the two series are not cointegrated, so M3 and Y3 are cointegrated.

4 Vector Autoregression (VAR)

Vector autoregression (VAR) is a stochastic process model used to capture the linear interdependencies among multiple time series.
On EViews is possible to estimates the Vector Autoregression model of the series M3 and Y3:

Vector Autoregression Estimates
Date: 12/04/19 Time: 20:41
Sample (adjusted): 1961M03 2008M12
Included observations: 574 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	M3	Y3
M3(-1)	0.964613	0.011295
	(0.05863)	(0.04799)
	[16.4521]	[0.23538]
M3(-2)	-0.016871	0.032214
	(0.05885)	(0.04817)
	[-0.28667]	[0.66883]
Y3(-1)	0.279668	1.107923
	(0.07258)	(0.05940)
	[3.85325]	[18.6510]
Y3(-2)	-0.241676	-0.163701
	(0.07192)	(0.05887)
	[-3.36013]	[-2.78089]
C	0.046720	0.108294
	(0.05849)	(0.04787)
	[0.79876]	[2.26221]
R-squared	0.968297	0.976174
Adj. R-squared	0.968074	0.976007
Sum sq. resids	143.7743	96.30850
S.E. equation	0.502672	0.411411
F-statistic	4344.716	5828.176
Log likelihood	-417.1524	-302.1548
Akaike AIC	1.470914	1.070226
Schwarz SC	1.508829	1.108141
Mean dependent	5.580406	6.357638
S.D. dependent	2.813284	2.656021
Determinant resid covariance (dof adj.)		0.021251
Determinant resid covariance		0.020882
Log likelihood		-518.5808
Akaike information criterion		1.841745
Schwarz criterion		1.917575
Number of coefficients		10

Figure 7: VAR estimates for M3 Y3

From the VAR (Figure 7) is possible to measure the lags length following the Schwarz criteria. With VAR lag Order selection criteria, we can get the information of the significant lags to fits our VAR model in the proper way.

On EViews VAR lag Order Selection Criteria using M3 and Y3 as endogenous variable.

VAR Lag Order Selection Criteria
Endogenous variables: M3 Y3
Exogenous variables: C
Date: 12/04/19 Time: 20:42
Sample: 1961M01 2008M12
Included observations: 568

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-2088.986	NA	5.402552	7.362626	7.377915	7.368592
1	-531.7271	3098.067	0.022769	1.893405	1.939273	1.911304
2	-518.3407	26.53718	0.022029*	1.860355*	1.936800*	1.890186*
3	-515.8703	4.879892	0.022148	1.865741	1.972765	1.907505
4	-511.6510	8.304882	0.022131	1.864968	2.002571	1.918665
5	-511.2319	0.822003	0.022412	1.877577	2.045758	1.943206
6	-506.4314	9.381215	0.022349	1.874759	2.073518	1.952320
7	-499.9305	12.65848*	0.022153	1.865952	2.095290	1.955447
8	-497.9464	3.849541	0.022311	1.873051	2.132966	1.974477

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 8: VAR Lag Order Selection Criteria

We follow the Schwarz criteria that suggests using 2 lags (but also the others suggest 2 lags) for the VAR model (Figure 8).

As we get the significant lags, we have a VAR model I(1). Since VAR can be applied if all the variables are I(0), then the model used is Vector Error Correction Model (VECM) if there exist at least one or more cointegration relationship among the variables.

5 Vector Error Correction Model (VECM)

The Vector Error Correction Model (VECM) is very useful by which to estimate the short-term effect for variables and the long run effect of the time series data.

5.1 Application of VECM

On EViews we define the VECM (Figure 9) with 2 lag intervals as suggested by Schwarz and specify intercept (no trend as we know the series have no trend) in CE e no intercept in VAR.

Vector Error Correction Estimates
Date: 12/04/19 Time: 20:44
Sample (adjusted): 1961M04 2008M12
Included observations: 573 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
M3(-1)	1.000000	
Y3(-1)	-0.999432	
	(0.05849)	
	[-17.0865]	
C	0.763868	
	(0.40292)	
	[1.89582]	
Error Correction:	**D(M3)**	**D(Y3)**
CointEq1	-0.053489	0.041666
	(0.02542)	(0.02075)
	[-2.10411]	[2.00787]
D(M3(-1))	0.033223	-0.012370
	(0.05966)	(0.04870)
	[0.55688]	[-0.25401]
D(M3(-2))	0.017856	0.030396
	(0.05902)	(0.04818)
	[0.30254]	[0.63092]
D(Y3(-1))	0.228973	0.151905
	(0.07189)	(0.05868)
	[3.18505]	[2.58853]
D(Y3(-2))	-0.090738	-0.122487
	(0.07257)	(0.05924)
	[-1.25040]	[-2.06777]
R-squared	0.050842	0.034094
Adj. R-squared	0.044158	0.027292
Sum sq. resids	143.9919	95.94822
S.E. equation	0.503495	0.411002
F-statistic	7.606347	5.012284
Log likelihood	-417.3584	-301.0542
Akaike AIC	1.474200	1.068252
Schwarz SC	1.512166	1.106218
Mean dependent	-0.004036	-0.003750
S.D. dependent	0.514994	0.416728
Determinant resid covariance (dof adj.)		0.021257
Determinant resid covariance		0.020887
Log likelihood		-517.7455
Akaike information criterion		1.852515
Schwarz criterion		1.951226
Number of coefficients		13

Figure 9: VEC Estimates outputs

5.2 Granger Causality test on VEC

The Granger Causality Test on VEC is an interesting test. By using this test, we can determine whether the short-term rates are the cause of the long-term and vice versa.

VEC Granger Causality/Block Exogeneity Wald Tests
Date: 12/09/19 Time: 13:56
Sample: 1961M01 2008M12
Included observations: 573

Dependent variable: D(M3)

Excluded	Chi-sq	df	Prob.
D(Y3)	11.90170	2	0.0026
All	11.90170	2	0.0026

Dependent variable: D(Y3)

Excluded	Chi-sq	df	Prob.
D(M3)	0.479200	2	0.7869
All	0.479200	2	0.7869

Figure 10: Granger Causality Tests outputs

From the test (Figure 10) if we consider as dependent variable M3, we cannot exclude past values of Y3 from the equation of M3 (since the p-value is lesser than 0.05). If we consider Y3 as dependent variable, we can exclude past value of M3 from the equation of Y3. Thus, Y3 Granger Cause M3 and M3 does not Granger Cause Y3. By that we can say that long-term interest rates Granger Cause the short ones which mean that first ones anticipate the second ones. Vice versa, short-term does not Granger Cause long ones.

6 Impulse response function (IRF)

The impulse response function is a method that can be used to determine the response of an endogenous variable toward a shock from the other variables.

We can define these functions on EViews specifying Cholesky decomposition method with no degree of freedom using order M3 Y3:

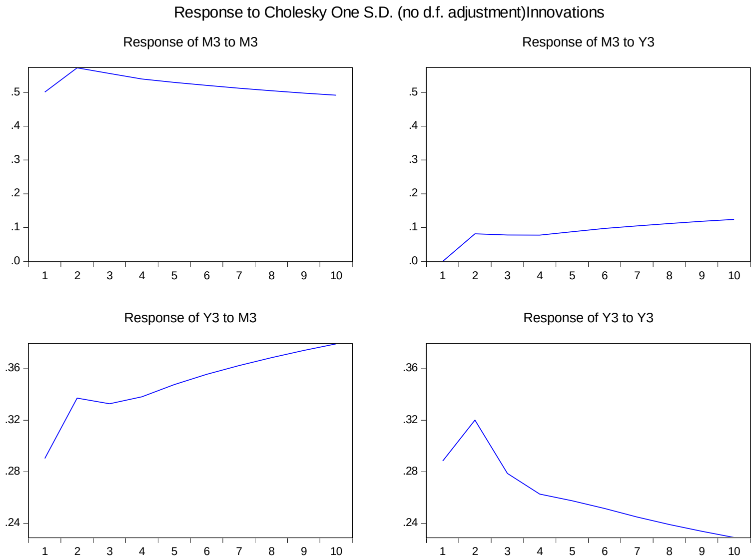

Figure 11: IRF M3 Y3

Considering the plot (Figure 11), is possible to see that M3 responds to itself for the first two lags but then it decreases. Also, M3 responds heavily to Y3 from the first lag to the second and then it stabilizes. Y3 responds heavily after the second lag to itself and then decreases significantly. Y3 responds to M3, so long-term maturity interest rates respond heavily to the short-term ones.

If we consider the IRF with order Y3 M3, we obtain the plots:

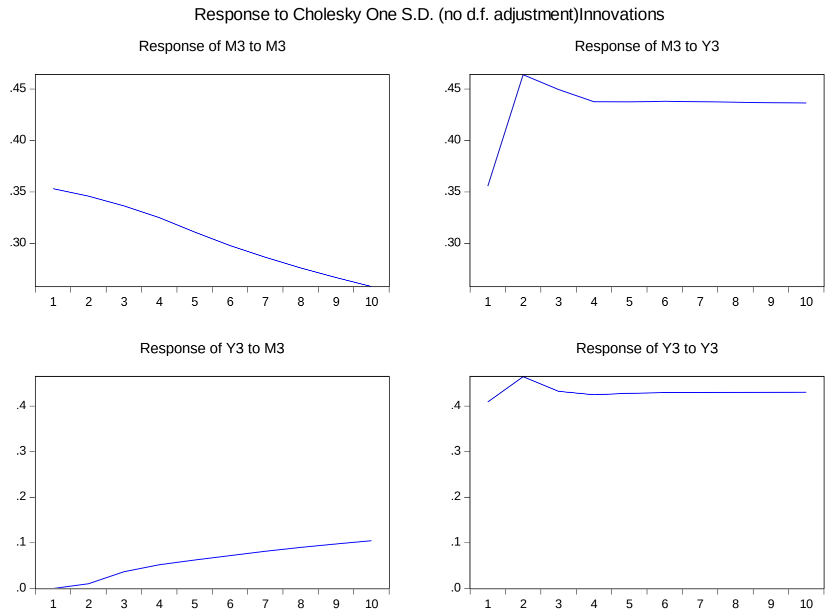

Figure 12: IRF Y3 M3

In this case (Figure 12), M3 does not respond to itself as lags increase and responds heavily to Y3 first two lags and then it stabilised. Y3 responds to itself on the first two lags and then it stabilised, while responds to M3 moderately.

7 Conclusion

We started from choosing two sample for our experiment between different short-term and long-term interest rates. We determined whether the two were I(0) or not using unit root test. As we get to the conclusion that the two were I(1), we check for cointegration using Engle-Granger test. Once we known that the two are cointegrated we generated the VAR model of the two to determine the lags to suite the model. Then we defined the VECM model for the variables, run the Granger Causality test and obtain the Impulse response functions.

Based on the discussion and results detailed before, we can conclude that the data for US dollar LIBOR interbank interest rates can be modelled by using Vector Error Correction Model (VECM). By using this model is possible to define a Granger Causality test and an Impulse Response Function (IRF). With the Granger Causality, we define that the long-term rates Granger Cause the short ones, while with the IRF we obtain different responses between short and long-term interest rates based on the Cholesky order.