Academic Year 2019-2020

B-74-3-B Time Series Econometrics

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EXERCISE SHEET 3

1.

We observed a process $\{Y_t\}_{t=-\infty}^{\infty}$ at $t = 1, ..., 4$, and recorded $y_1 = -0.4$, $y_2 = 0.8$, $y_3 = 0.6$, $y_4 = -0.2$.

i. Compute the objective function $\sum_{t=1}^{T} \varepsilon_t^2 (\theta)$, where $\varepsilon_t (\theta) = Y_t - \theta\varepsilon_{t-1} (\theta)$, assuming $\varepsilon_0 = 0$ for the values $\theta = 0.5$, $\theta = -0.5$, $\theta = 0$.

ii. What estimation techniques requires you to compute $\sum_{t=1}^{T} \varepsilon_t^2 (\theta)$?

iii. What is your estimate of θ?

2.

Open the file ARMAs.wf1 in eviews.

i. Inspect the correlogram of the time series y. Explain why a *MA(1)* model is a good choice for this time series.

ii. Estimate a *MA(1)* model for y (include the constant in the estimation equation). Make a note of the estimated coefficients and of their estimated standard errors. Using eviews, test the assumption $H_0 : \{\theta_0 = 0.9\}$ with a Wald test. Make a note of the value of the test statistic and explain wether the null hypothesis is rejected.

iii. Estimate a *MA(2)* model for y (include the constant in the estimation equation). Make a note of the estimated coefficients and of their estimated standard errors. Using eviews test the assumption $H_0 : \{\theta_{0;1} = 0.9, \theta_{0;2} = 0\}$ with a Wald test. Make a note of the value of the test statistic and explain wether the null hypothesis is rejected.

iv. Compare the results of parts ii. and iii. and explain what may have caused any difference that you observed.

3.

Let $\{Y\}_{t=-\infty}^{\infty}$ be a stationary and invertible process generated by the ARMA(p, q)

$$Y_t = \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + \varepsilon_t + ... + \theta_q \varepsilon_{t-q},$$

where ε_t is independently and identically distrubuted with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Let $\ln lik(p, q)$ indicate the maximised log-likelihood for the generic ARMA(p, q).

Suggest the orders p, q if

$$\begin{aligned} \ln lik(1,0) &= -248.6914 \\ \ln lik(0,1) &= -257.1481 \\ \ln lik(1,1) &= -248.6750 \\ \ln lik(0,2) &= -251.3668 \\ \ln lik(2,0) &= -247.8323 \end{aligned}$$

have been computed for the process Y_t, given that $T = 200$.

4.

Let $\{Y\}_{t=-\infty}^{\infty}$ be a stationary and invertible process generated by the ARMA(p, q)

$$Y_t = \phi_1 Y_{t-1} + ... + \phi_p Y_{t-p} + \varepsilon_t + ... + \theta_q \varepsilon_{t-q},$$

where ε_t is independently and identically distrubuted with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Suppose that the following ARMA(1,1) has been estimated, using a sample of 200 observations,

$$Y_t = 0.4Y_{t-1} + \widehat{\varepsilon}_t - 0.1\widehat{\varepsilon}_{t-1},$$

where $\widehat{\varepsilon}_t$ are the residuals. Let

$$r_j = \frac{\frac{1}{T}\sum_{t=j+1}^{T} \widehat{\varepsilon}_t \widehat{\varepsilon}_{t-j}}{\frac{1}{T}\sum_{t=1}^{T} \widehat{\varepsilon}_t^2}$$

be the j^{th} sample autocorrelation of those residuals, and assume that you computed $r_1 = 0.05$, $r_2 = -0.07$, $r_3 = 0.1$.

Does the ARMA(1,1) constitute an acceptable approximation of the given process?

5. Open the file ARMAs.wf1 in eviews. i. Estimate models MA(1) and MA(2) for series Y_t and take note of the maximised likelihoods. Which model has higher likelihood? Propose a model for Y_t between MA(1) and MA(2). ii. Estimate the MA(1) model, and test for residual autocorrelation using the Portmanteau test.