UNIVERSITÀ DEGLI STUDI DI MILANO
Dipartimento di Economia, Management
e Metodi Quantitativi

B-74-3-B Time Series Econometics

Specimen Paper #1

Time allowed: 90 Minutes

Authorized material:

- Non-programmable calculator (for personal use only)
- One A4 page (one-sided) of handwritten, personal notes (for personal use only)

Material provided at the time of the exam:

- Probability Tables as on ARIEL

During the exam:

- Put your student card in a visible place to facilitate identity control
- No questions will be answered
- You are not allowed to leave the room

The Exam is divided in TWO parts: Questions 1 to 4 are Short Questions; Questions 5 and 6 are Long Questions. Answers to Short Questions are worth 12.5% of the final mark per question; Answers to Long Questions are worth 25% of the final mark per question.

Full marks may be obtained by complete answers to ALL six questions.

A complete answer to a question should include a clear statement of all the necessary steps in the argument, together with any assumptions and working.

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Question 1 (12.5% of total mark).

Denote $\{X_t\}_{t=-\infty}^{\infty}$ as the process generated by the ARMA(2,1) model

$$X_t = X_{t-1} - 0.25X_{t-2} + u_t + u_{t-1}$$

where $\{u\}_{t=-\infty}^{\infty}$ is a white noise process with $E(u_t) = 0$, $E(u_t^2) = \sigma^2$.

1.1 Check if $\{X_t\}_{t=-\infty}^{\infty}$ is stationary.

1.2 Compute the Impulse Response Function of $\{X_t\}_{t=-\infty}^{\infty}$ for the first 3 lags.

Question 2 (12.5% of total mark).

Denote $\{X_t\}_{t=-\infty}^{\infty}$ as the process generated by the model

$$X_t = 1 + 0.25t + u_t$$

where $\{u\}_{t=-\infty}^{\infty}$ is the AR(2) process

$$u_t = u_{t-1} - 0.5u_{t-2} + \varepsilon_t$$

For $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ a white noise process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Suppose that we observed $X_{10} = 3$ and $X_9 = 3.05$. Compute the forecast $\hat{X}_{11|10,9}$

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Question 3 (12.5% of total mark).

Let $\{Y_t\}_{t=-\infty}^{\infty}$ be a stationary and invertible process generated by the ARMA(p,q) model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + +\theta_q \varepsilon_{t-q}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.

Suppose that we have a time series $\{Y_1, \dots, Y_{100}\}'$ (i.e., T = 100) and that we estimated, by maximum likelihood, five models: an AR(1), a MA(1), an AR(2), a MA(2) and an ARMA(1,1).
For each model, let *lnlik*, BIC and AIC indicate the maximised log-likelihood, the Bayes information criterion and the Akaike information criterion, respectively. These are

Model	Lnlik	BIC	AIC
ARMA(1,1)	-127.405	264.020	258.810
AR(2)	-128.394	265.998	260.788
MA(2)	-129.761	268.732	263.522
MA(1)	-138.390	281.385	278.780
AR(1)	-129.312	263.229	260.624

3.1) Give the formulae for the AIC and for the BIC.

3.2) Why is it not advisable to use both criteria, and it is recommended to choose only one criterion instead? If you were to choose between the AIC and the BIC, how would you motivate your choice?

3.3) In view of your answer above, what model would you recommend?

Question 4 (12.5% of total mark).

4.1) What does it mean to say that a process is stationary?

4.2) What does it mean to say that a process is integrated of order 0, I(0)?

4.3) Provide an example of a process that is stationary but it is not I(0).

Question 5 (25% of total mark).

Denote $\{X_t\}_{t=-\infty}^{\infty}$ as the process generated by the model

$$X_t = \alpha + \rho X_{t-1} + u_t \quad (1)$$

where $\{u_t\}_{t=-\infty}^{\infty}$ is an independent process with $E(u_t) = 0$, $E(u_t^2) = \sigma^2$, if $t > 0$; $X_t = 0$ if $t \leq 0$.
Suppose that you are concerned that the process has a unit root, and the true data generating process is, in fact, $X_t = X_{t-1} + u_t$. You therefore decided to test the hypothesis $H_0 = \{\rho = 1\}$ using the Dickey and Fuller test.

5.1) Show that equation (1) can be written as

$$\Delta X_t = \alpha + (\rho - 1)X_{t-1} + u_t \ (2)$$

5.2) Explain advantages and disadvantages of testing $H_0 = \{\rho = 1\}$ using regression (2) as opposed to the restricted regression

$$\Delta X_t = (\rho - 1)X_{t-1} + u_t \qquad (3)$$

5.3) Suppose now that $\{u\}_{t=-\infty}^{\infty}$ is the (stationary) AR (3) process

$$u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \phi_3 u_{t-3} + \varepsilon_t$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$.
Explain how your test should be modified to test for a unit root in this case. For a complete answer, you should explain both how your regression of choice is modified to account for the dependence in $\{u\}_{t=-\infty}^{\infty}$, and how the critical values are affected by this.

(Question 5 continues on the next page)

5.4) We have data on US quarterly inflation at quarterly frequency over the period 1985 – 2010 (included).
Using eviews we tested

Null Hypothesis: INFL has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=4)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.092440	0.2482
Test critical values:	1% level	-3.494378	
	5% level	-2.889474	
	10% level	-2.581741	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(TRUE)
Method: Least Squares
Sample: 1985Q1 2010Q4
Included observations: 104

Variable	Coefficient	Std. Error	t-Statistic	Prob.
INFL(-1)	-0.229036	0.109459	-2.092440	0.0390
D(INFL(-1))	-0.592641	0.124049	-4.777469	0.0000
D(INFL(-2))	-0.453123	0.119946	-3.777713	0.0003
D(INFL(-3))	-0.240537	0.097991	-2.454693	0.0158
C	0.488868	0.274206	1.782849	0.0777

R-squared	0.416638	Mean dependent var	-0.019863
Adjusted R-squared	0.393068	S.D. dependent var	1.295774
S.E. of regression	1.009484	Akaike info criterion	2.903639
Sum squared resid	100.8867	Schwarz criterion	3.030773
Log likelihood	-145.9892	Hannan-Quinn criter.	2.955144
F-statistic	17.67647	Durbin-Watson stat	1.933521
Prob(F-statistic)	0.000000		

Pointing to information provided in this output, explain if the null hypothesis is rejected.

5.5) In view of the outcome above, would you model inflation in levels or in first differences?

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Question 6 (25% of total mark).

Let$\{Y_t\}_{t=-\infty}^{\infty}$be the process generated by the model

$$Y_t = \varepsilon_t + \theta_0 \varepsilon_{t-1}$$

where $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent process with $E\ (\varepsilon_t\ )\ =\ 0$, $E\ (\varepsilon_t^2\ ) =\ \sigma_0^2$, so that $E\ (Y_t^2\ ) = \sigma_0^2(1+\theta_0^2)$, $E\left(Y_t\ \ Y_{t+1}\right) = \sigma_0^2\theta_0$, $E\left(Y_t\ \ Y_{t+1}\right) = 0$ when j > 1
(note: you do not need to derive these results).

Let $Y = \{Y_1, \dots, Y_T\}'$, and $E(Y\ Y') = \Omega_0$, so that the joint density of Y computed at the points $y = \{y_1, \dots, y_T\}'$, is

$$f_Y(y) = (2\pi)^{-\frac{T}{2}} |\Omega_0|^{-\frac{1}{2}} \exp\{-\frac{1}{2} y' |\Omega_0|^{-\frac{1}{2}} y\}$$

6.1 Write the likelihood function for a generic vector of unknown parameters $\beta\ \ = (\ \theta\ \ ,\sigma\ \ ^2)'$ and for a given set of observations $y = \{y_1, \dots, y_T\}'$ for variables of $Y = \{Y_1, \dots, Y_T\}'$, and define the exact maximum likelihood estimate of $\beta_0 = (\ \theta_0, \sigma_0{}^2)'$

6.2 The conditional density of $Y_t|\varepsilon_{t-1}$ is

$$f_{Y_t|\varepsilon_t}(y_t) = (2\pi\sigma_0^2)^{(-1/2)} \exp\{-\frac{1}{2}\frac{(y_t - \theta_0\varepsilon_{t-1})^{2'}}{\sigma_0^2}\}$$

Assume that we know $\varepsilon_0 = 0$ and θ_0. Explain how it is possible to compute ε_t using the observations $y_1, \dots, y_T$ in this case.

6.3 Explain how this can be used to compute the density of $\{Y_1, \dots, Y_T\}'$ conditional on $\varepsilon_0 = 0$ and define the conditional maximum likelihood estimate of β_0.

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