Decision methods and models

(Prof. Roberto Cordone)

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Available time: 2 hours and 30 minutes

Note: the answers can be given in Italian or English at will; to avoid penalisations, clarify all assumptions and motivate all computational steps.

Exercise 1 - Given a decision problem with impact set $F = \{a, b, c, d, e, f\}$ and preference relation $\Pi = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, c), (b, d), (c, b), (c, c), (c, d), (d, d), (e, b), (e, c), (e, d), (e, e), (f, b), (f, c), (f, d), (f, f)\}$

a) briefly define the concepts of *impact* and *indicator* in a decision problem;

b) explain the role of the preference relation in the problem;

c) list the main properties enjoyed by Π given above, explain whether it is an order relation (and which) and what consequences this has for the decision problem;

d) compute the associated indifference relation Ind_Π.

Exercise 2 - Given the following mathematical programming problem:

$$\begin{aligned} \min f(x) &= x_2^2 + x_1 - 4x_2 \\ g_1(x) &= x_1^2 + x_2^2 - 6x_1 - 4x_2 + 9 \leq 0 \\ g_2(x) &= x_2 - 2 \leq 0 \end{aligned}$$

a) represent it graphically;

b) determine the nonregular points (if any exist);

c) determine the candidate points according to Karush-Kuhn-Tucker's conditions, and in particular the global minimum point(s).

Exercise 3 - Briefly describe the *inverse transformation method* to enumerate the Paretian solutions, specifying its advantages and disadvantages.

Apply the method to the following problem:

$$\begin{aligned} \max f_1 &= x_1 + x_2 \\ \max f_2 &= x_1 - x_2 \\ x_1^2 + x_2^2 &\leq 1 \\ x_1 &\leq 0 \end{aligned}$$

Exercise 4 - Briefly describe the following aspects of the Analytic Hierarchy Process (*AHP*): a) the use of qualitative scales; b) the use of pairwise comparisons; c) the use of hierarchical attribute structures.

Derive a weight vector from the following pairwise comparison matrix or explain why it is not possible:

$$\tilde{\Lambda} = \begin{bmatrix} 1 & 1/12 & 1/4 & 1/20 \\ 12 & 1 & 3 & 3/5 \\ 4 & 1/3 & 1 & 1/5 \\ 20 & 5/3 & 5 & 1 \end{bmatrix}$$

Exercise 5 - Given the following evaluation matrix, whose values represent **benefits**

$u_{a\omega}$	ω_1	ω_2	ω_3	ω_4
a_1	70	20	-100	30
a_2	80	0	-20	-40
a_3	-10	70	-10	-20

a) explain the meaning of symbols a, ω, $u_{a\omega}$;

b) choose an alternative with the *Laplace criterium* (equiprobability);

c) show the alternatives chosen with the *Hurwicz criterium* as the *pessimism coefficient* α varies.

Exercise 6 - Considering the problems in conditions of uncertainty:

a) briefly describe the *expected value criterium* and its formal defects;

b) formally define the concept of *lottery* according to Von Neumann and Morgenstern;

c) briefly define the *continuity axiom* of Von Neumann and Morgenstern's theory;

d) briefly define the *substitution axiom* of Von Neumann and Morgenstern's theory.

Exercise 7 - Given the following *payoff matrix* for a two-player game:

	a	b	c
a	(3,8)	(6,9)	(2,10)
b	(4,5)	(9,2)	(8,6)
c	(1,7)	(5,0)	(10,2)

determine the dominated strategies and the Nash equilibria (if any exist).

Briefly describe the concept of *stag hunt* game.

Briefly define the concept of *ideal marriage* game.

Exercise 8 - Considering Arrow's theory on democracy:

a) briefly define the concept of *social welfare function* as a method to aggregate preferences;

b) briefly describe the *universal domain axiom* in Arrow's theory;

c) discuss the possible consequences of rejecting this axiom;

d) briefly define the concept of *minimal decisive set* for a pair of impacts according to Arrow's theory.