

Homework - Sept 27, 2020 - Advanced Multivariate Statistics

Instructor: Giancarlo Manzi

Master in Data Science & Economics

NOT MANDATORY!

Must be done alone. Maximum score: 1 point.

If you have questions, please contact the instructor for assistance.

Issued: Sunday, September 27th, 2020

Due: **Wednesday, September 30th, 2020**

Part a: (*Checking if a function is a proper density and using the moment generating function. Solutions should be found via R*).

Consider the following function:

$$f(x) = \begin{cases} \frac{1}{7} & 0 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

- (i) Check whether it is a proper density.
- (ii) Plot it.
- (iii) If it is a density, what is the random variable having this density?
- (iv) Compute the moment generating function for this r.v. evaluated at 1.

Hints: for (i) use the `integrate()` R function; for (ii) use the `curve()` R function ; for (iv) use the `distr6` library.

Part b: (*Finding distributions of transformations of a multivariate normal. Solutions should be found by hand*).

Let $\mathbf{x} \sim N_4(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

- (i) Find the distribution of $(x_1, x_4)'$.
- (ii) Find the distribution of $x_2 - x_4$.

Part c: (*Sample mean vectors and covariance matrices. Solutions should be found via R*).

Find the sample mean vector and the sample covariance matrix for the following observa-

tions

| x_1 | x_2 | x_3 |
|-------|-------|-------|
| 12 | 10 | 8 |
| 11 | 8 | 11 |
| 8 | 5 | 11 |
| 14 | 11 | 13 |
| 10 | 7 | 6 |

A script containing the R code and a MS word file with solutions should be sent to **giancarlo.manzi@unimi.it**.

Please give your student ID number in your message.

Suggested Reading:

R package `distr6`. In particular:

- https://alan-turing-institute.github.io/distr6/articles/webs/constructing_a_distribution.html about how to construct a distribution object.
- https://alan-turing-institute.github.io/distr6/articles/webs/statistical_methods.html about how to get statistical methods from distributions.