(5) B is a basis for V because $\mathbf{v_1}$ and $\mathbf{v_1}$ are linearly independent (easy to check). If $\mathbf{v}$ is in V, then the following vector equation is consistent:

$$\begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix} = c_1 \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

The scalars c_1 and c_2, if they exist, are the $V-$coordinates of $\mathbf{v}$. Row operations show that

$$\begin{bmatrix} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus $c_1 = 2$, $c_2 = 3$ (geometrically $\mathbf{v}$ belongs to the plane in $\mathbb{R}^3$ generated by the vectors $\mathbf{v_1}$ and $\mathbf{v_2}$).

(6) Reduce A to echelon form

$$A \sim \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & -7 \\ 0 & -6 & 4 & 14 & -20 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix} \sim \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & -7 \\ 0 & 0 & 0 & 4 & -6 \\ 0 & 0 & 0 & -10 & 15 \end{bmatrix} \sim \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & -7 \\ 0 & 0 & 0 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The matrix A has 3 pivot columns, so $rank(A) = 3$.

(7) The Leontief's model is the following, $\mathbf{x} \in \mathbb{R}^3$,

$$\begin{cases} x_1 = 0.5x_1 + 0.4x_2 + 0.2x_3 + 50 \\ x_2 = 0.2x_1 + 0.3x_2 + 0.1x_3 + 30 \\ x_3 = 0.1x_1 + 0.1x_2 + .3x_3 + 20 \end{cases} \Leftrightarrow \begin{cases} 0.5x_1 - 0.4x_2 - 0.2x_3 = 50 \\ -0.2x_1 + 0.7x_2 - 0.1x_3 = 30 \\ -0.1x_1 - 0.1x_2 + 0.7x_3 = 20 \end{cases}$$

We have to solve a linear system, but .... is there a solution? is it unique? how can we calculate it?

Reduce the augmented matrix of the linear system to echelon form:

$$\tilde{A} = \begin{bmatrix} 0.5 & -0.4 & -0.2 & 50 \\ -0.2 & 0.7 & -0.1 & 30 \\ -0.1 & -0.1 & 0.7 & 20 \end{bmatrix} \sim \ldots \sim \begin{bmatrix} 0.5 & -0.4 & -0.2 & 50 \\ 0 & 0.54 & -0.18 & 50 \\ 0 & 0 & 0.6 & 140/3 \end{bmatrix}$$

which corresponds to a consistent system with only one solution (that we can compute with the backward substitution):

$$\mathbf{x} = \begin{bmatrix} 6100/27 \\ 3200/27 \\ 700/9 \end{bmatrix}$$